Multi-granulation method for information fusion in multi-source decision information system

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A B S T R A C T
Most of the existing multi-source fusion methods are to choose the most reliable information from the multi-source information system to form a single-source information system. Obviously, this process is accompanied by information loss. In order to solve this problem, the multi-granulation method of information fusion in multi-source decision information system is studied in this paper. Firstly, decision support characteristic function and decision related characteristic function are constructed. Secondly, a pair of aggregation operators, including fixed aggregation operator and possible aggregation operator, is defined through two characteristic functions. Meanwhile, the two cases when thresholds \(\alpha\) and \(\beta\) take special values are discussed. Finally, the relevant properties of aggregation operators in different situations are proposed and proved. What is more, two groups of comparative experiments are carried out to illustrate the effect of the aggregation operators. The experimental results show that the proposed multi-source fusion method can always find a set of thresholds \((\alpha, \beta)\), which makes the fusion effect better than the mean fusion.

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1. Introduction

Many complex information systems came into being with the advent of the era of big data. How to discover knowledge and extract rules from these complex information systems is an urgent problem to be solved. Rough set theory (RST) [1] provides us with a good idea. The RST was firstly proposed by researchers represented by polish mathematician Pawlak in 1982, and the first monograph [2] on RST was published in 1992. The RST [3,4] is an effective tool for dealing with incomplete information. At the same time, the RST is also a very effective method to deal with complex information systems. Traditional RST had been extended from different levels in order to solve different problems. The classical RST was established during the equivalence relation in the domain. However, the relation we study often fails to meet the standard of equivalence relation due to the lack of information and other reasons. For this reason, generalized rough sets based on general relations [5–11] were researched. Zhang and Yang proposed the three-way group decisions in interval-valued decision-theoretic rough sets by combining the aggregating inclusion measures [12].

As a kind of complex information system, multi-source decision information system (MsDIS) has been widely concerned by scholars. However, the rough set models mentioned above cannot solve the problem of knowledge discovery in MsDIS.

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Because the equivalence relation in classical RST is considered as a granularity and a partition of universe can be regarded as a granularity structure from the perspective of granular computing [13,14]. Therefore, the classical RST is formed in a single granularity space under an equivalence relation. It can be seen that the above expansion of classical RST are carried out in single granularity space. However, there are multiple information tables in a MsDIS, and each information table corresponds to a granularity structure. At this time, the traditional RST and its extended models are no longer applicable.

In order to apply RST more widely to complex information systems in real life, Qian et al. proposed multi-granulation rough sets [15,16]. In recent years, many scholars have also done a lot of researches on multi-granulation rough sets [17–23]. The multi-granulation rough set was studied by Yang et al. from the perspective of sensitive cost [24,25]. Hu et al. studied the dynamic approximation updating of multi-granulation rough sets by using matrix method, and realized the decision-making under dynamic conditions [26]. Li et al. analyzed the relation between multi-granulation rough sets and concept lattices from the perspective of rule acquisition [27]. Qian et al. combined with the idea of probability theory to propose a multi-granulation decision-theoretic rough set model [28]. Lin et al. replaced the classical equivalence relation with the neighborhood relationship to expand the multi-granulation rough set theory [29]. Zhang et al. studied the local multi-granulation DTRS in the order information system [30]. Lin et al. proposed a multi-threshold tolerance relation-based multi-granulation rough sets for incomplete interval-valued decision information systems [31]. Yu et al. proposed a double-quantitative decision making theory in a multi-granulation approximate space [32]. Xu et al. studied multigranulation fuzzy rough set [33,34]. Zhan and Xu proposed a method of solving the multi-criterion group decision [35]. Qian et al. proposed a multiple different thresholds based generalized multigranulation sequential three-way decision model [36]. Sang et al. proposed Generalized Multi-granulation double-quantitative DTRS in multi-source information system [37].

In recent years, information fusion [38–42], as an interesting method to deal with multi-source information system (MsIS), has been widely noticed. Most of the existing fusion methods transform MsIS into single-source information system (SsIS) and then carry out knowledge discovery and rule extraction. Xu et al. considered information from the perspective of triangular fuzzy granular, and realized the selection of information sources through the internal and external confidence [43]. Huang et al. studied the dynamic fusion of interval value data sets starting from trapezoidal fuzzy granular [44]. Guo et al. proposed the reduction method of multi-source information systems: consistent reduction and reduction after fusion [45]. Because of the ambiguity and incompleteness of the source, Xu et al. used conditional entropy to select information sources, and then merged the multi-source information table into a table [46,47]. The principle of those information fusion method is similar to that of the mean fusion represented by case 1 in Fig. 1. The common feature of these fusion methods is that before knowledge discovery or rule extraction, the most reliable information in the MsIS is selected and integrated into a SsIS through different methods. It is worth noting that there is information loss in multi-source fusion by these methods. Therefore, how to discover knowledge from MsDIS without information loss is the motivation of this paper.

In 2018, Wei and Liang discussed the overview of information fusion from the perspective of RST [48]. It is known from literature [48] that multi-granulation rough set is an effective tool for information fusion. Lin et al. studied information fusion by combining multi-granulation and evidence theory [49]. Che et al. studied the information fusion and related digital characteristics of multi-source information system from the perspective of multi-granulation [50]. Similarly, this paper also studies the information fusion method of MsDIS from the perspective of multi-granulation rough set. We consider the equivalence relation formed by all conditional attributes in each information source as a granularity, and then q information sources form q granularity. Then the aggregation operators are constructed through multi-granulation rough set. The main idea of this paper is illustrated in a more concise manner, which is shown in case 2 in Fig. 1. The research results show that
we can also classify objects without merging multi-source decision information tables into one table. In addition, one can see from the research results that the higher the correlation between sources, the better the effect of our proposed fusion method.

The main contributions of this paper are as follows. First, we define the decision support characteristic function and decision related characteristic function to describe the inclusion relation and the intersection non-empty relation between equivalent class and concept in each information source. Second, three groups of aggregation operators are built through two characteristic functions. Meanwhile, the properties of aggregation operators are proposed and proved. Third, we propose two comparison algorithms, and experiment with 9 data sets on UCI shows that the fusion method proposed in this paper can realize direct knowledge discovery without information loss from MsDIS.

The rest of this paper is divided into the following sections. In section 2, we mainly review related works like the approximate accuracy of concepts, multi-granulation rough sets, and multi-source decision information systems. In section 3, firstly, decision support characteristic function and decision related characteristic function are constructed; secondly, three sets of aggregation operators are defined through these two functions; finally, the properties of the aggregation operators are discussed. In order to prove the validity of our proposed aggregation operators, we download 9 data sets in the machine learning database and conduct a series of experiments in section 4. In section 5, the conclusion of this paper was given.

2. Preliminaries

As we study the multi-granulation approach of information fusion in multi-source decision information systems, we briefly review the basic concepts related to rough sets, support characteristic function, multi-granulation rough sets and multi-source decision information systems in this section.

2.1. Pawlak's rough sets

In general, we use a quad \( I = (U, AT, V, F) \) to represent an information system, where \( U \) is a non-empty finite set of objects, called the universe; \( AT \) is a non-empty finite set of attributes; \( V = \bigcup_{a \in AT} V_a \) is the domain of the attribute \( a \); \( F = \{ f : U \times AT \rightarrow V \} \) is the set of information functions. \( f(x, a) \in V_a \) \( (x \in U, \ a \in AT) \). Let \( X, Y \) be two non-empty classical sets, \( X \times Y = \{(x, y) : x \in X, y \in Y\} \) is Cartesian product of \( X \) and \( Y \). For any \( R \subseteq X \times Y \), \( R \) is called the binary relation from \( X \) to \( Y \), abbreviated as a relation.

**Definition 2.1.** [51] Let \( R \subseteq X \times X \), \( R \) is an equivalence relation on \( X \) if \( R \) satisfy:

1. reflexive, i.e. \( \forall x \in X, (x, x) \in R \);
2. symmetric, i.e. \( \forall x, y \in X, \text{if } (x, y) \in R, \text{then } (y, x) \in R \);
3. transitive, i.e. \( \forall x, y, z \in X, \text{if } (x, y) \in R \text{ and } (y, z) \in R, \text{then } (x, z) \in R \).

If \( R \) is an equivalence relation on \( X \), for any \( x \in X \), the equivalent class represented by \( x \) is denoted as \( [x]_R = \{y \in X : (x, y) \in R\} \), abbreviated as an equivalent class.

**Definition 2.2.** [51] Let \( R \) be an equivalence relation on the domain \( U \), and \( (U, R) \) is an approximate space. For a target set \( X (X \subseteq U) \), the lower and upper approximation sets of the target set \( X \) for \( R \) are defined as follows:

\[
\overline{R}(X) = \bigcup \{x \in U : [x]_R \subseteq X\}, \quad \underline{R}(X) = \bigcup \{x \in U : [x]_R \cap X \neq \emptyset\}.
\]

If \( \overline{R}(X) = \overline{R}(X) \), then \( X \) is definable or accurate for \( R \); otherwise, \( X \) is rough about \( R \).

**Definition 2.3.** [51] Let \( (U, R) \) be an approximate space, \( X \subseteq U \ (X \neq \emptyset) \). The approximate accuracy and roughness of the target set \( X \) are defined as follows:

\[
\varrho_R(X) = \frac{|\overline{R}(X)|}{|\underline{R}(X)|}, \quad \rho_R(X) = 1 - \varrho_R(X).
\]

They are used to measure the uncertainty of \( X \) through the numerical characteristics of \( X \). \( |A| \) represents the number of elements in the set \( A \).

For a decision system \( I = (U, AT, V, F) \), where \( AT = C \cup D \), \( C \) is a conditional attribute set, and \( D \) is a decision attribute set. \( U/D = \{D_1, D_2, \ldots, D_m\} \) is the division of the domain \( U \) on the decision attribute set, and \( A \) is a subset of the conditional attribute set. The lower and upper approximation sets of the partition \( U/D \) are defined as follows:

\[
\overline{A}(U/D) = \overline{A}(D_1) \cup \overline{A}(D_2) \cup \cdots \cup \overline{A}(D_m), \quad \underline{A}(U/D) = \underline{A}(D_1) \cup \underline{A}(D_2) \cup \cdots \cup \underline{A}(D_m).
\]

The definition of approximate accuracy and roughness for \( U/D \) is shown as follows:
\[
Q_A(U/D) = \frac{\sum_{D_j \in U/D} |A(D_j)|}{\sum_{D_j \in U/D} |\overline{A}(D_j)|}, \quad \rho_A(U/D) = 1 - Q_A(U/D).
\]

2.2. Support characteristic function and generalized multi-granulation rough set model

The support characteristic function is defined by the relation between the equivalence class and the concept set, then the generalized multi-granulation rough set model is defined by the support characteristic function.

Definition 2.4. [52] Let \( I = (U, AT, V, F) \) be an information system, \( A_i \subseteq AT, i = 1, 2, \ldots, s \ (s \leq 2^{|AT|}) \). For any \( X (X \subseteq U) \), the support characteristic function of \( x \) for \( X \) is defined as follows:

\[
S^A_i(x) = \begin{cases} 1, & |x|A_i \subseteq X, i \leq 2^{|AT|}. \\ 0, & \text{otherwise}, \end{cases}
\]

The support characteristic function \( S^A_i(x) \) is used to describe the inclusion relation between equivalence class \(|x|A_i\) and concept \( X \), which indicates whether object \( x \) accurately supports \( X \) by \( A_i \).

Definition 2.5. [52] Let \( I = (U, AT, V, F) \) be an information system, \( A_i \subseteq AT, i = 1, 2, \ldots, s \ (s \leq 2^{|AT|}) \), \( \beta \in (0.5, 1) \), \( S^A_i(x) \) is the support characteristic function of \( x \) for \( X \). For any \( X \subseteq U \), the lower approximation and upper approximation of \( X \) for \( \sum A_i \) are defined as follows:

\[
\frac{GM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \sum_{i=1}^{s} S^A_i(x) \geq \beta \right\}, \quad \frac{OM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \sum_{i=1}^{s} (1 - S^A_i(x)) < 1 - \beta \right\}.
\]

The target set \( X \) is definable if \( GM_{\sum A_i}^{\sum A_i} = OM_{\sum A_i}^{\sum A_i} \), otherwise, \( X \) is a rough set. We denote this generalized multi-granulation rough set model as GMGRS, and \( \beta \) is the information level for \( \sum A_i \).

2.3. Pessimistic and optimistic multi-granulation rough set models

When the domain is divided by multiple equivalence relations, combined with the idea of single-granulation rough set, the multi-granulation rough set is divided into pessimistic multi-granulation rough set and optimistic multi-granulation rough set.

Definition 2.6. [52] Let \( I = (U, AT, V, F) \) be an information system, \( A_i \subseteq AT, i = 1, 2, \ldots, s \ (s \leq 2^{|AT|}) \). For any \( X \subseteq U \), the pessimistic lower approximation and pessimistic upper approximation of \( X \) for \( \sum A_i \) are defined as follows:

\[
\frac{PM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \land_{i=1}^{s} (|x|A_i \subseteq X) \right\}, \quad \frac{OM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \lor_{i=1}^{s} (|x|A_i \cap X \neq \emptyset) \right\},
\]

where “\( \lor \)” and “\( \land \)” denote “or” and “and”, respectively. If you consider pessimistic multi-granulation from the perspective of support characteristic function, the formula (7) has another form as follows:

\[
\frac{PM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \sum_{i=1}^{s} S^A_i(x) \geq \beta \right\}, \quad \frac{OM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \sum_{i=1}^{s} (1 - S^A_i(x)) < 1 - \beta \right\}.
\]

Definition 2.7. [52] Let \( I = (U, AT, V, F) \) be an information system, \( A_i \subseteq AT, i = 1, 2, \ldots, s \ (s \leq 2^{|AT|}) \). For any \( X \subseteq U \), the optimistic lower approximation and optimistic upper approximation of \( X \) for \( \sum A_i \) are defined as follows:

\[
\frac{OM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \lor_{i=1}^{s} (|x|A_i \subseteq X) \right\}, \quad \frac{OM_{\sum A_i}}{\sum A_i}(X) = \left\{ x \in U \mid \land_{i=1}^{s} (|x|A_i \cap X \neq \emptyset) \right\}.
\]
where $\lor$ and $\land$ denote “or” and “and”, respectively. If you consider optimistic multi-granulation from the perspective of support characteristic function, the formula (9) has another form as follows:

$$\sum_{i=1}^{s} \frac{S_{X}^{A_{i}}(x)}{s} > 0, \quad \sum_{i=1}^{s} \frac{1 - S_{X}^{A_{i}}(x)}{s} \geq 1. \quad (10)$$

### 2.4. Multi-source decision information systems

The same structure of multi-source information system was obtained by acquiring information about the same object and the same attribute from different information sources. The definition of the multi-source information system is given as follows.

**Definition 2.8.** [49] A multi-source information system is composed by a plurality of quads in the form of $IS_{i} = (U, AT, V_{i}, F_{i})$, where $U$ represents the entirety of the object, $AT$ represents the attribute set, $V_{i}$ represents the range of attribute values under source $i$, and $F_{i}$ represents the corresponding relation between the object and the feature under the source $i$. In general, A multi-source information system (MsIS) is expressed as follows:

$$M_{i}IS = \{IS_{1}, IS_{2}, \ldots, IS_{q}\}. \quad (11)$$

Similarly, a multi-source decision information system (MsDIS) is composed by multiple single information systems in the form of $DIS_{i} = (U, AT, V_{i}, F_{i}, DT, D_{i}, G_{i})$, where the meanings of $U, AT, V_{i}$ and $F_{i}$ are consistent with those described in the MsIS, $DT$ represents the decision attribute set, $D_{i}$ represents the range of the decision attribute value under source $i$, and $G_{i}$ represents the corresponding relation between the object and the decision attribute under source $i$. In general, the MsDIS is expressed as follows:

$$M_{S}DIS = \{DIS_{1}, DIS_{2}, \ldots, DIS_{q}\}. \quad (12)$$

### 3. Multi-granulation method for information fusion in multi-source decision information system

The traditional information fusion method regards the MsDIS as an information box, and then fuses the information box into an information table by means of fuzzy granules and conditional entropy. The traditional fusion process is shown in Fig. 2. Finally, the fused information table is used for decision-making and rule extraction. In order to discover knowledge directly from multi-source decision data sets, the fusion of multi-source information was constructed from the view of multi-granulation in this section.

#### 3.1. Fixed aggregation operator and possible aggregation operator of MsDIS

In this subsection, inspired by the idea of RST, we first define the decision support characteristic function and the decision related characteristic function, and then construct the information aggregation operators (the fixed aggregation operator and
possible aggregation operator) of the MsDIS from the perspective of generalized multi-granulation, and finally we propose and prove the related properties of aggregation operators.

**Definition 3.1.** Given a multi-source decision information system $M_5DIS = \{DIS_1, DIS_2, \ldots, DIS_q\}$, where $DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i)$ $(i = 1, 2, \ldots, q)$, $U/D = \{D_1, D_2, \ldots, D_m\}$, for each $D_j \in U/D$ $(j = 1, 2, \ldots, m)$, the decision support characteristic function and decision related characteristic function of $x$ for $D_j$ are defined as follows:

$$D^{DIS_i}_{D_j}(x) = \begin{cases} 1, &[x]_{DIS_i} \subseteq D_j, \\ 0, &\text{otherwise}. \end{cases}$$

$$DR^{DIS_i}_{D_j}(x) = \begin{cases} 1, &[x]_{DIS_i} \cap D_j \neq \emptyset, \\ 0, &\text{otherwise}. \end{cases}$$

(13)

The decision support characteristic function $D^{DIS_i}_{D_j}(x)$ is used to describe the inclusion relation between equivalence class $[x]_{DIS_i}$ and concept $D_j$, which indicates whether object $x$ accurately supports $D_j$ by $DIS_i$. The decision related characteristic function $DR^{DIS_i}_{D_j}(x)$ is used to describe the intersection non-empty relation between equivalence class $[x]_{DIS_i}$ and concept $D_j$, which indicates whether object $x$ possibly supports $D_j$ by $DIS_i$.

**Definition 3.2.** Let $M_5DIS = \{DIS_1, DIS_2, \ldots, DIS_q\}$ be a MsDIS, where $DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i)$ $(i = 1, 2, \ldots, q)$, $\alpha, \beta$ are two thresholds which satisfied $\alpha + \beta = 1$ and $\alpha \in (0, 1], \beta \in [0, 1)$. $U/D = \{D_1, D_2, \ldots, D_m\}$, $D^{DIS_i}_{D_j}(x)$ is the decision support characteristic function of $x$ for $D_j$, $DR^{DIS_i}_{D_j}(x)$ is the decision related characteristic function of $x$ for $D_j$. For any $D_j \in U/D$ $(j = 1, 2, \ldots, m)$, the fixed aggregation operator and possible aggregation operator of MsDIS are defined as follows:

$$\bigotimes_{i=1}^{q} DIS_i(D_j)_{\alpha} = \left\{ x \in U : \frac{\sum_{i=1}^{q} D^{DIS_i}_{D_j}(x)}{q} \geq \alpha \right\}, \quad \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta} = \left\{ x \in U : \frac{\sum_{i=1}^{q} DR^{DIS_i}_{D_j}(x)}{q} > \beta \right\}.$$  

(14)

**Definition 3.3.** The definition of fusion approximate accuracy and fusion roughness when dividing $U/D$ at thresholds $(\alpha, \beta)$ is shown as follows:

$$Q^{M_{SD}}(U/D)_{(\alpha, \beta)} = \frac{\sum_{D_j \in U/D} \bigotimes_{i=1}^{q} DIS_i(D_j)_{\alpha}}{\sum_{D_j \in U/D} \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta}}, \quad \rho^{M_{SD}}(U/D)_{(\alpha, \beta)} = 1 - Q^{M_{SD}}(U/D)_{(\alpha, \beta)}.$$  

(15)

**Property 3.1.** Let $M_5DIS = \{DIS_1, DIS_2, \ldots, DIS_q\}$ be a MsDIS, where $DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i)$ $(i = 1, 2, \ldots, q)$. $\alpha, \beta$ are two thresholds which satisfied $\alpha + \beta = 1$ and $\alpha \in (0, 1], \beta \in [0, 1)$. $U/D = \{D_1, D_2, \ldots, D_m\}$, for any $D_j \in U/D$ $(j = 1, 2, \ldots, m)$, the following properties of aggregation operators are true.

1. $\bigotimes_{i=1}^{q} DIS_i(D_j)_{\alpha} \subseteq D_j \subseteq \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta}$;
2. $\bigotimes_{i=1}^{q} DIS_i(\emptyset)_{\alpha} = \bigoplus_{i=1}^{q} DIS_i(\emptyset)_{\beta} = \emptyset$;
3. $\bigotimes_{i=1}^{q} DIS_i(U)_{\alpha} = \bigoplus_{i=1}^{q} DIS_i(U)_{\beta} = U$;
4. $\forall D_1, D_j \in U/D$, $\bigotimes_{i=1}^{q} DIS_i(D_1 \cap D_j)_{\alpha} \subseteq \bigotimes_{i=1}^{q} DIS_i(D_1)_{\alpha} \cap \bigotimes_{i=1}^{q} DIS_i(D_j)_{\alpha}$,
5. $\bigotimes_{i=1}^{q} DIS_i(D_1 \cap D_j)_{\beta} \subseteq \bigotimes_{i=1}^{q} DIS_i(D_1)_{\beta} \cap \bigotimes_{i=1}^{q} DIS_i(D_j)_{\beta}$;
6. $\forall D_1, D_j \in U/D$, $\bigotimes_{i=1}^{q} DIS_i(D_1 \cup D_j)_{\alpha} \geq \bigotimes_{i=1}^{q} DIS_i(D_1)_{\alpha} \cup \bigotimes_{i=1}^{q} DIS_i(D_j)_{\alpha}$,
7. $\bigotimes_{i=1}^{q} DIS_i(D_1 \cup D_j)_{\beta} \geq \bigotimes_{i=1}^{q} DIS_i(D_1)_{\beta} \cup \bigotimes_{i=1}^{q} DIS_i(D_j)_{\beta}$. 
Proof. (1) For any $x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$, we can know \( \frac{\sum_{i=1}^{q} \text{DS}^{\text{DIS}_i(x)}_D}{q} \geq \alpha \). Because of $\alpha \in (0, 1]$, so \( \exists i \leq q, \text{s.t.} [x]_{\text{DIS}_i} \subseteq D_j \). Known by the arbitrariness of $x$, $\bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha \subseteq D_j$. For any $x \in D_j$, we can know $[x]_{\text{DIS}_i} \cap D_j \neq \emptyset$. Thus $\frac{\sum_{i=1}^{q} \text{DR}^{\text{DIS}_i(x)}_D}{q} = 1 > \beta$, $x \in \bigoplus_{i=1}^{q} \text{DIS}_i(D_{ij})_\beta$. Known by the arbitrariness of $x$, $D_j \subseteq \bigoplus_{i=1}^{q} \text{DIS}_i(D_{ij})_\beta$. Therefore, the property (1) has been proved.

(2) Through the definition of decision support characteristic function and decision related characteristic function, we can gain $\text{DS}^{\alpha}_{\text{DIS}_i(x)} = 0$, $\text{DR}^{\alpha}_{\text{DIS}_i(x)} = 0$. So we can obtain the following two equations:

\[
\bigotimes_{i=1}^{q} \text{DIS}_i(\emptyset)_{\alpha} = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} \text{DS}^{\alpha}_{\text{DIS}_i(x)}}{q} = \frac{\sum_{i=1}^{q} 0}{q} = 0 \geq \alpha \right. \right\} = \emptyset,
\]

\[
\bigoplus_{i=1}^{q} \text{DIS}_i(\emptyset)_{\beta} = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} \text{DR}^{\alpha}_{\text{DIS}_i(x)}}{q} = \frac{\sum_{i=1}^{q} 0}{q} = 0 > \beta \right. \right\} = \emptyset.
\]

(3) Through the definition of decision support characteristic function and decision related characteristic function, we can obtain $\text{DS}^{\alpha}_{U} = 1$, $\text{DR}^{\alpha}_{U} = 1$. So we can gain the following two equations:

\[
\bigotimes_{i=1}^{q} \text{DIS}_i(U)_{\alpha} = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} \text{DS}^{\alpha}_{\text{DIS}_i(x)}}{q} = \frac{\sum_{i=1}^{q} 1}{q} = 1 \geq \alpha \right. \right\} = U,
\]

\[
\bigoplus_{i=1}^{q} \text{DIS}_i(U)_{\beta} = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} \text{DR}^{\alpha}_{\text{DIS}_i(x)}}{q} = \frac{\sum_{i=1}^{q} 1}{q} = 1 > \beta \right. \right\} = U.
\]

(4) $\forall x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij} \cap D_{ij})_\alpha$, we have

\[
\sum_{i=1}^{q} \frac{\text{DS}^{\text{DIS}_i(x)}_{D_{ij} \cap D_{ij}}}{q} = \frac{\sum_{i=1}^{q} \text{DS}^{\text{DIS}_i(x)}_{D_{ij} \cap D_{ij}}}{q} \geq \alpha.
\]

So $\sum_{i=1}^{q} \frac{\text{DS}^{\text{DIS}_i(x)}_{D_{ij}}}{q} \cdot \sum_{i=1}^{q} \frac{\text{DS}^{\text{DIS}_i(x)}_{D_{ij}}}{q} \geq \alpha$. Thus $\frac{\sum_{i=1}^{q} \text{DS}^{\text{DIS}_i(x)}_{D_{ij}}}{q} \geq \alpha$ and $\sum_{i=1}^{q} \frac{\text{DS}^{\text{DIS}_i(x)}_{D_{ij}}}{q} \geq \alpha$, i.e. $x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$ and $x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$. Therefore $x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha \cap \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$. Known by the arbitrariness of $x$, $\forall D_{ij} \subseteq U/D$, $\bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij}) \subseteq \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha \cap \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$. The same we can prove $\bigoplus_{i=1}^{q} \text{DIS}_i(D_{ij} \cap D_{ij})_\beta \subseteq \bigoplus_{i=1}^{q} \text{DIS}_i(D_{ij})_\beta \cap \bigoplus_{i=1}^{q} \text{DIS}_i(D_{ij})_\beta$.

(5) $\forall x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha \cup \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$, i.e. $x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$ or $x \in \bigotimes_{i=1}^{q} \text{DIS}_i(D_{ij})_\alpha$, by definition we have $\sum_{i=1}^{q} \frac{\text{DS}^{\alpha}_{D_{ij} \cup D_{ij}}}{q} \geq \alpha$ or $\frac{\sum_{i=1}^{q} \text{DS}^{\alpha}_{D_{ij} \cup D_{ij}}}{q} \geq \alpha$. What’s more, $\frac{\sum_{i=1}^{q} \text{DS}^{\alpha}_{D_{ij}}}{q} \geq \left( \frac{\sum_{i=1}^{q} \text{DS}^{\alpha}_{D_{ij}}}{q} \cup \sum_{i=1}^{q} \text{DR}^{\alpha}_{D_{ij}}(x) \right) \geq \alpha$, so $\frac{\sum_{i=1}^{q} \text{DS}^{\alpha}_{D_{ij \cup D_{ij}}}}{q} \geq \alpha$, i.e. $x \in$
\( q \bigotimes_{i=1}^{q} DIS_i(D_1 \cup D_j)_{a_2} \). Known by the arbitrariness of \( x, \forall D_i, D_j \in U/D, \bigotimes_{i=1}^{q} DIS_i(D_1 \cup D_j)_{a_2} \supseteq \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_2} \cup \bigotimes_{i=1}^{q} DIS_i(D_j)_{a_2} \). Similarly, we can prove \( \bigoplus_{i=1}^{q} DIS_i(D_1 \cup D_j)_{\beta} \supseteq \bigoplus_{i=1}^{q} DIS_i(D_1)_{\beta} \cup \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta} \).

Property 3.2. Let \( MS DIS = \{DIS_1, DIS_2, \ldots, DIS_q\} \) be a MsDIS, where \( DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i) (i = 1, 2, \ldots, q) \). \( \alpha, \beta \) are two thresholds which satisfied \( \alpha + \beta = 1 \) and \( \alpha \in (0, 1), \beta \in [0, 1), U/D = \{D_1, D_2, \ldots, D_m\} \). For different levels of information \((\alpha_1 \leq \alpha_2)\), the following properties are established.

(1) \( \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_2} \subseteq \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_1} \);

(2) \( \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta_1} \subseteq \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta_2} \).

Proof. (1) For any \( x \in \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_2} \), we can know \( \frac{\sum_{i=1}^{q} DS_D^{DIS}(x)}{q} \geq \alpha_2 \). Because of \( \alpha_1 \leq \alpha_2 \), then \( \frac{\sum_{i=1}^{q} DS_D^{DIS}(x)}{q} \geq \alpha_1 \), thus \( x \in \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_1} \). Known by the arbitrariness of \( x, \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_2} \subseteq \bigotimes_{i=1}^{q} DIS_i(D_1)_{a_1} \).

(2) For any \( x \in \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta_1} \), we can know \( \frac{\sum_{i=1}^{q} DR_D^{DIS}(x)}{q} > \beta_1 \). Because of \( \alpha + \beta = 1 \), so \( \beta_1 = 1 - \alpha_1, \beta_2 = 1 - \alpha_2 \), and \( \alpha_1 \leq \alpha_2 \), then \( \beta_1 \geq \beta_2 \), therefore \( \frac{\sum_{i=1}^{q} DR_D^{DIS}(x)}{q} > \beta_2 \), i.e. \( x \in \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta_2} \). Known by the arbitrariness of \( x, \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta_1} \subseteq \bigoplus_{i=1}^{q} DIS_i(D_j)_{\beta_2} \).

3.2. Pessimistic fixed aggregation operator and pessimistic possible aggregation operator of MsDIS

In this subsection, the pessimistic fixed aggregation operator and pessimistic possible aggregation operator of MsDIS are constructed from the angle of pessimistic multi-granulation rough set.

Definition 3.4. Let \( MS DIS = \{DIS_1, DIS_2, \ldots, DIS_q\} \) be a MsDIS, where \( DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i) (i = 1, 2, \ldots, q) \). \( U/D = \{D_1, D_2, \ldots, D_m\} \), for any \( D_j \in U/D \) \((j = 1, 2, \ldots, m)\), the pessimistic fixed aggregation operator and pessimistic possible aggregation operator of MsDIS are defined as follows:

\[
q \bigotimes_{i=1}^{q} DIS_i(D_j) = \{x \in U \mid \bigwedge_{i=1}^{q} ([x]DIS_i \subseteq D_j)\} = \left\{x \in U \mid \frac{\sum_{i=1}^{q} DS_D^{DIS}(x)}{q} = 1\right\},
\]

\[
q \bigoplus_{i=1}^{q} DIS_i(D_j) = \{x \in U \mid \bigvee_{i=1}^{q} ([x]DIS_i \cap D_j \neq \emptyset)\} = \left\{x \in U \mid \frac{\sum_{i=1}^{q} DR_D^{DIS}(x)}{q} > 0\right\}.
\]

Definition 3.5. The definition of pessimistic fusion approximate accuracy and pessimistic fusion roughness for dividing \( U/D \) is shown as follows:

\[
\varrho_{M MSD}(U/D)^P = \frac{\sum_{D_j \in U/D} \left| q \bigotimes_{i=1}^{q} DIS_i(D_j) \right|}{\sum_{D_j \in U/D} \left| q \bigoplus_{i=1}^{q} DIS_i(D_j) \right|}, \quad \rho_{M MSD}(U/D)^P = 1 - \varrho_{M MSD}(U/D)^P.
\]

Property 3.3. Let \( MS DIS = \{DIS_1, DIS_2, \ldots, DIS_q\} \) be a MsDIS, where \( DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i) (i = 1, 2, \ldots, q) \). \( U/D = \{D_1, D_2, \ldots, D_m\} \), for any \( D_j \in U/D \) \((j = 1, 2, \ldots, m)\), the following properties are true.
Proof. (1) For any $x \in \bigotimes_{i=1}^{q} DIS_i(D_j)$, we can know $\frac{\sum_{i=1}^{q} DS^\Theta_{\delta_{D_j}}(x)}{q} = 1$, i.e. $\forall i \leq q$, s.t. $x \in DIS_i \subseteq D_j$. Known by the arbitrariness of $x$, $\bigotimes_{i=1}^{q} DIS_i(D_j) \subseteq D_j$. Similarly, we can prove $D_j \subseteq \bigoplus_{i=1}^{q} DIS_i(D_j)$. Therefore, $\bigotimes_{i=1}^{q} DIS_i(D_j) \subseteq D_j \subseteq \bigoplus_{i=1}^{q} DIS_i(D_j)$.

(2) Through the definition of decision support characteristic function and decision related characteristic function, we can obtain $DS^\Theta_{\delta_{D_j}}(x) = 0$, $DR^\Theta_{\delta_{D_j}}(x) = 0$. So we can gain the following two equations:

$$\bigotimes_{i=1}^{q} DIS_i(\emptyset) = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} DS^\Theta_{\delta_{D_j}}(x)}{q} = \frac{\sum_{i=1}^{q} 0}{q} = 0 \geq 1 \right. \right\} = \emptyset,$$

$$\bigoplus_{i=1}^{q} DIS_i(\emptyset) = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} DR^\Theta_{\delta_{D_j}}(x)}{q} = \frac{\sum_{i=1}^{q} 0}{q} = 0 > 0 \right. \right\} = \emptyset.$$

(3) Through the definition of decision support characteristic function and decision related characteristic function, we can obtain $DS^\Omega_{\delta_{D_j}}(x) = 1$, $DR^\Omega_{\delta_{D_j}}(x) = 1$. So we can obtain the following two equations:

$$\bigotimes_{i=1}^{q} DIS_i(U) = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} DS^\Omega_{\delta_{D_j}}(x)}{q} = \frac{\sum_{i=1}^{q} 1}{q} = 1 \geq 1 \right. \right\} = U,$$

$$\bigoplus_{i=1}^{q} DIS_i(U) = \left\{ x \in U \left| \frac{\sum_{i=1}^{q} DR^\Omega_{\delta_{D_j}}(x)}{q} = \frac{\sum_{i=1}^{q} 1}{q} = 1 > 0 \right. \right\} = U.$$

(4) $\forall x \in \bigotimes_{i=1}^{q} DIS_i(D_i \cap D_j) \iff \frac{\sum_{i=1}^{q} DS^\Omega_{\delta_{D_j}}(x)}{q} = \frac{\sum_{i=1}^{q} DS^\Omega_{\delta_{D_j}}(x) \wedge DS^\Omega_{\delta_{D_j}}(x)}{q} = 1 \iff \frac{\sum_{i=1}^{q} DS^\Omega_{\delta_{D_j}}(x)}{q} = 1$ and $\frac{\sum_{i=1}^{q} DS^\Omega_{\delta_{D_j}}(x)}{q} = 1 \iff x \in \bigotimes_{i=1}^{q} DIS_i(D_i)$ and $x \in \bigotimes_{i=1}^{q} DIS_i(D_j) \iff x \in \bigotimes_{i=1}^{q} DIS_i(D_i) \cap \bigotimes_{i=1}^{q} DIS_i(D_j)$. Known by the arbitrariness of $x$, $\forall D_i, D_j \in U/D$, $\bigotimes_{i=1}^{q} DIS_i(D_i \cap D_j) = \bigotimes_{i=1}^{q} DIS_i(D_i) \cap \bigotimes_{i=1}^{q} DIS_i(D_j)$.
For any \( x \in \bigoplus_{i=1}^{q} P DIS_i(D_i \cap D_j) \), we have
\[
\sum_{i=1}^{q} D^P DIS_i(D_i) (x) < 0, \quad \sum_{i=1}^{q} D^P DIS_i(D_j) (x) > 0 \quad \text{and} \quad \sum_{i=1}^{q} D^P DIS_i(D_i) (x) > 0.
\]
So
\[
\sum_{i=1}^{q} D^P DIS_i(D_i) (x) > 0 \quad \text{and} \quad \sum_{i=1}^{q} D^P DIS_i(D_j) (x) > 0.
\]
\( x \in \bigoplus_{i=1}^{q} P DIS_i(D_i) \) and \( x \in \bigoplus_{i=1}^{q} P DIS_i(D_j) \), so \( x \in \bigoplus_{i=1}^{q} P DIS_i(D_i) \bigcup \bigoplus_{i=1}^{q} P DIS_i(D_j) \). Known by the arbitrariness of \( x \), \( \forall D_i, D_j \in U/D, \bigoplus_{i=1}^{q} P DIS_i(D_i \cap D_j) \subseteq \bigoplus_{i=1}^{q} P DIS_i(D_i) \bigcup \bigoplus_{i=1}^{q} P DIS_i(D_j) \).

(5) \( \forall x \in \bigoplus_{i=1}^{q} P DIS_i(D_i) \bigcup \bigoplus_{i=1}^{q} P DIS_i(D_j), x \in \bigoplus_{i=1}^{q} P DIS_i(D_i) \) or \( x \in \bigoplus_{i=1}^{q} P DIS_i(D_j) \),
\[
\frac{\sum_{i=1}^{q} D^P DIS_i(D_i) (x)}{q} = 1 \text{ or } \frac{\sum_{i=1}^{q} D^P DIS_i(D_j) (x)}{q} = 1.
\]
What's more,
\[
\frac{\sum_{i=1}^{q} D^P DIS_i(D_i) (x)}{q} \geq \frac{\sum_{i=1}^{q} D^P DIS_i(D_j) (x)}{q} \geq 1, \quad \text{so} \quad \frac{\sum_{i=1}^{q} D^P DIS_i(D_i) (x)}{q} \geq 1, \quad x \in \bigoplus_{i=1}^{q} P DIS_i(D_i \cup D_j).
\]
Known by the arbitrariness of \( x \), \( \forall D_i, D_j \in U/D, \bigoplus_{i=1}^{q} P DIS_i(D_i \cup D_j) \supseteq \bigoplus_{i=1}^{q} P DIS_i(D_i) \bigcup \bigoplus_{i=1}^{q} P DIS_i(D_j) \).

3.3. Optimistic fixed aggregation operator and optimistic possible aggregation operator of MsDIS

Like the previous subsection, the optimistic fixed aggregation operator and optimistic possible aggregation operator of MsDIS are constructed from the angle of optimistic multi-granulation rough set.

**Definition 3.6.** Let \( M_5 DIS = \{DIS_1, DIS_2, \ldots, DIS_q\} \) be a MsDIS, where \( DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i) (i = 1, 2, \ldots, q) \), \( U/D = \{D_1, D_2, \ldots, D_m\} \), for any \( D_j \in U/D \) \( (j = 1, 2, \ldots, m) \), the optimistic fixed aggregation operator and optimistic possible aggregation operator of MsDIS are defined as follows:

\[
\bigotimes_{i=1}^{q} O DIS_i(D_j) = \left\{ x \in U \mid \bigvee_{i=1}^{q} ([x]_{DIS_i} \subseteq D_j) \right\} = \left\{ x \in U \mid \frac{\sum_{i=1}^{q} D^P DIS_i(D_i) (x)}{q} > 0 \right\},
\]

\[
\bigoplus_{i=1}^{q} O DIS_i(D_j) = \left\{ x \in U \mid \bigwedge_{i=1}^{q} ([x]_{DIS_i} \cap D_j \neq \emptyset) \right\} = \left\{ x \in U \mid \frac{\sum_{i=1}^{q} D^P DIS_i(D_i) (x)}{q} = 1 \right\}.
\]

**Definition 3.7.** The definition of optimistic fusion approximate accuracy and optimistic fusion roughness for dividing \( U/D \) is shown as follows:

\[
Q_{M_5DIS}(U/D)^O = \frac{\sum_{D_j \in U/D} \bigotimes_{i=1}^{q} O DIS_i(D_j)}{\sum_{D_j \in U/D} \bigoplus_{i=1}^{q} O DIS_i(D_j)}, \quad P_{M_5DIS}(U/D)^O = 1 - Q_{M_5DIS}(U/D)^O.
\]

**Property 3.4.** Let \( M_5 DIS = \{DIS_1, DIS_2, \ldots, DIS_q\} \) be a MsDIS, where \( DIS_i = (U, AT, V_i, F_i, DT, D_i, G_i) (i = 1, 2, \ldots, q) \), \( U/D = \{D_1, D_2, \ldots, D_m\} \), for any \( D_j \in U/D \) \( (j = 1, 2, \ldots, m) \), the following properties are true.

(1) \( \bigotimes_{i=1}^{q} O DIS_i(D_j) \subseteq D_j \subseteq \bigoplus_{i=1}^{q} O DIS_i(D_j) \);
(2) \( \bigotimes_{i=1}^{q} DIS_i(\emptyset) = \bigoplus_{i=1}^{q} DIS_i(\emptyset) = \emptyset; \)

(3) \( \bigotimes_{i=1}^{q} DIS_i(U) = \bigoplus_{i=1}^{q} DIS_i(U) = U; \)

(4) \( \forall D_i, D_j \in U / D, \bigotimes_{i=1}^{q} DIS_i(D_i \cap D_j) \subseteq \bigotimes_{i=1}^{q} DIS_i(D_i) \cap \bigotimes_{i=1}^{q} DIS_i(D_j); \)

(5) \( \forall D_i, D_j \in U / D, \bigotimes_{i=1}^{q} DIS_i(D_i \cup D_j) \supseteq \bigotimes_{i=1}^{q} DIS_i(D_i) \cup \bigotimes_{i=1}^{q} DIS_i(D_j). \)

**Proof.** (1) For every \( x \in \bigotimes_{i=1}^{q} DIS_i(D_j) \), we can know \( \sum_{i=1}^{q} DS_{DIS_i}^{O}(x) > 0 \). So \( \exists i \leq q, \text{ s.t. } [x]_{DIS_i} \subseteq D_j \). Known by the arbitrariness of \( x, \bigotimes_{i=1}^{q} DIS_i(D_j) \subseteq D_j \). For each \( x \in D_j \), we can know \([x]_{DIS_i} \cap D_j \neq \emptyset\). So \( \sum_{i=1}^{q} DS_{DIS_i}^{O}(x) = 1, x \in \bigoplus_{i=1}^{q} DIS_i(D_j) \). Known by the arbitrariness of \( x, D_j \subseteq \bigoplus_{i=1}^{q} DIS_i(D_j) \). Thus, the property (1) is proved.

(2) Through the definition of decision support characteristic function and decision related characteristic function, we can obtain \( DS_{DIS_i}^{O}(x) = 0, DR_{DIS_i}^{O}(x) = 0 \). So we can have the following two equations:

\[
\bigotimes_{i=1}^{q} DIS_i(\emptyset) = \left\{ x \in U \left| \sum_{i=1}^{q} DS_{DIS_i}^{O}(x) = 0 \right. \right\} = \emptyset,
\]

\[
\bigoplus_{i=1}^{q} DIS_i(\emptyset) = \left\{ x \in U \left| \sum_{i=1}^{q} DR_{DIS_i}^{O}(x) = 0 \right. \right\} = \emptyset.
\]

(3) Through the definition of decision support characteristic function and decision related characteristic function, we can obtain \( DS_{DIS_i}^{U}(x) = 1, DR_{DIS_i}^{U}(x) = 1 \). So we can gain the following two equations:

\[
\bigotimes_{i=1}^{q} DIS_i(U) = \left\{ x \in U \left| \sum_{i=1}^{q} DS_{DIS_i}^{U}(x) = 1 \right. \right\} = U,
\]

\[
\bigoplus_{i=1}^{q} DIS_i(U) = \left\{ x \in U \left| \sum_{i=1}^{q} DR_{DIS_i}^{U}(x) = 1 \right. \right\} = U.
\]

(4) For every \( x \in \bigotimes_{i=1}^{q} DIS_i(D_i \cap D_j) \), we can have \( \sum_{i=1}^{q} DS_{DIS_i}^{O}(x) \wedge \sum_{i=1}^{q} DS_{DIS_i}^{O}(x) > 0 \). Because of

\[
\frac{\sum_{i=1}^{q} DS_{DIS_i}^{O}(x) \wedge DS_{DIS_i}^{O}(x)}{q} \geq \frac{\sum_{i=1}^{q} DS_{DIS_i}^{O}(x) \wedge DS_{DIS_i}^{O}(x)}{q} > 0,
\]

\[
\frac{\sum_{i=1}^{q} DS_{DIS_i}^{O}(x) \wedge DS_{DIS_i}^{O}(x)}{q} > 0.
\]
Table 1: A multi-source decision information system.

<table>
<thead>
<tr>
<th>DIS₁</th>
<th>DIS₂</th>
<th>DIS₃</th>
<th>DIS₄</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>a₁</td>
<td>a₂</td>
<td>a₃</td>
<td>a₄</td>
<td>d</td>
</tr>
<tr>
<td>o₁</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>a₂</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>a₃</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>a₄</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Thus \( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} > 0 \) and \( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} > 0 \), i.e. \( x \in \bigotimes_{i=1}^{q} DIS_i(D_i) \) and \( x \in \bigotimes_{i=1}^{q} DIS_i(D_j) \). So \( x \in \bigotimes_{i=1}^{q} DIS_i(D_i) \cap \bigotimes_{i=1}^{q} DIS_i(D_j) \).

Known by the arbitrariness of \( x \), \( \forall D_i, D_j \in U/D, \bigotimes_{i=1}^{q} DIS_i(D_i) \cap \bigotimes_{i=1}^{q} DIS_i(D_j) \). Similarly, we can prove \( \bigotimes_{i=1}^{q} DIS_i(D_i) \cap \bigotimes_{i=1}^{q} DIS_i(D_j) \).

(5) For any \( x \in \bigotimes_{i=1}^{q} DIS_i(D_i) \cup \bigotimes_{i=1}^{q} DIS_i(D_j) \), \( x \in \bigotimes_{i=1}^{q} DIS_i(D_i) \) or \( x \in \bigotimes_{i=1}^{q} DIS_i(D_j) \), \( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} > 0 \) or \( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} > 0 \).

What’s more, \( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} \geq \left( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} \right) \cup \left( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} \right) > 0 \), so \( \frac{\sum_{i=1}^{q} D S_{D i}^{D S i}(x)}{q} > 0 \), \( x \in \bigotimes_{i=1}^{q} DIS_i(D_i) \cup \bigotimes_{i=1}^{q} DIS_i(D_j) \). Known by the arbitrariness of \( x \), \( \forall D_i, D_j \in U/D, \bigotimes_{i=1}^{q} DIS_i(D_i) \cup \bigotimes_{i=1}^{q} DIS_i(D_j) \). Similarly, we can prove \( \bigotimes_{i=1}^{q} DIS_i(D_i) \cup \bigotimes_{i=1}^{q} DIS_i(D_j) \).

Property 3.5. Let \( M_S DIS = \{ DIS_1, DIS_2, \ldots, DIS_q \} \) be a MsDIS, where \( DIS_i = (U, AT, V_i, F_i, D_i, G_i) (i = 1, 2, \ldots, q) \). \( \alpha, \beta \) are two thresholds which satisfied \( \alpha + \beta = 1 \) and \( \alpha \in (0, 1) \), \( \beta \in [0, 1) \). \( U/D = \{ D_1, D_2, \ldots, D_m \} \), three kinds of fusion approximation accuracy (Eq. (15), Eq. (18), Eq. (21)) satisfies the following properties.

\[
Q_{MsDIS}(U/D)^\alpha \leq Q_{MsDIS}(U/D)_{(\alpha, \beta)} \leq Q_{MsDIS}(U/D)^0.
\]

Proof. It’s easy to prove by Definition 3.2-3.7 and Property 3.2.

3.4. Case study

There are 10 plots of land that need to be tested to determine whether they are suitable for growing crops. If only the soil of a certain point in a piece of land is selected for testing, the conclusion may be greatly deviated. So we chose to collect the soil in four directions of each piece of land for testing. Finally, experts in the industry will judge whether the land is suitable for growing crops by neutralizing the results of the four directions. We use the soil in four directions as four information sources to obtain a MsDIS. In this MsDIS, our research object is \( U = \{ a_1, a_2, \ldots, a_{10} \} \), each element in \( U \) represents a piece of land. The elements in \( AT = \{ a_1, a_2, a_3, a_4 \} \) represent four different microelement in soil, \( DT = \{ d \} \) is the decision attribute, the role here is to determine whether the land is suitable for growing crops. If \( d = 1 \), the land is suitable for planting crops; if \( d = 0 \), the land is not suitable for planting crops. More detailed information about this MsDIS is shown in Table 1.

First we should calculate the equivalence class of each object under each source.

The equivalence class of the object in \( DIS_1 \):

\[\{a_1|DIS_1 = \{a_1\}, a_2|DIS_1 = \{a_2\}, a_3|DIS_1 = \{a_3, a_7\}, a_4|DIS_1 = \{a_4\}, a_5|DIS_1 = \{a_5\}, a_6|DIS_1 = \{a_6\}, a_7|DIS_1 = \{a_3, a_7\}, a_8|DIS_1 = \{a_8, a_{10}\}, a_9|DIS_1 = \{a_9\}, a_{10}|DIS_1 = \{a_8, a_{10}\}\].

The equivalence class of the object in \( DIS_2 \):

\[\{a_1|DIS_2 = \{a_1, a_2\}, a_2|DIS_2 = \{a_1, a_2\}, a_3|DIS_2 = \{a_3\}, a_4|DIS_2 = \{a_4, a_{10}\}, a_5|DIS_2 = \{a_5\}, a_6|DIS_2 = \{a_6\}, a_7|DIS_2 = \{a_7\}, a_8|DIS_2 = \{a_8\}, a_9|DIS_2 = \{a_9\}, a_{10}|DIS_2 = \{a_4, a_{10}\}\].
The equivalence class of the object in $DI S_3$:

$[01]_{DIS_3} = [01, 02, 07], [02]_{DIS_3} = [01, 02, 07], [03]_{DIS_3} = [03, 05], [04]_{DIS_3} = [04, 010], [05]_{DIS_3} = [03, 05], [06]_{DIS_3} = [06], [07]_{DIS_3} = [01, 02, 07], [08]_{DIS_3} = [08], [09]_{DIS_3} = [09], [10]_{DIS_3} = [04, 010].$

The equivalence class of the object in $DIS_4$:

$[01]_{DIS_4} = [01], [02]_{DIS_4} = [02, 07], [03]_{DIS_4} = [03], [04]_{DIS_4} = [04, 010], [05]_{DIS_4} = [05], [06]_{DIS_4} = [06], [07]_{DIS_4} = [02, 07], [08]_{DIS_4} = [08], [09]_{DIS_4} = [09], [10]_{DIS_4} = [04, 010].$

Division under attribute $d$: $U/D = \{(01, 02, 04, 06, 08), (03, 05, 07, 09, 010)\}$.

Then calculate the decision support characteristic function and decision related characteristic function of the object for each decision class under each source:

The decision support characteristic function and decision related characteristic function for $D_1 = [01, 02, 04, 06, 08]$ are shown as follows:

$$
\begin{align*}
&\mathcal{D}_{DS_{DIS_1}}(01) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(01) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(01) = 0, \quad \mathcal{D}_{DS_{DIS_4}}(01) = 1, \quad \mathcal{D}_{DS_{DIS_1}}(02) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(02) = 1, \\
&\mathcal{D}_{DS_{DIS_3}}(04) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(04) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(05) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(05) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(06) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(06) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(07) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(07) = 1, \\
&\mathcal{D}_{DS_{DIS_3}}(08) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(08) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(09) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(09) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(10) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(10) = 1, \\
&\mathcal{D}_{DS_{DIS_3}}(11) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(11) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(12) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(12) = 1.
\end{align*}
$$

The decision support characteristic function and decision related characteristic function for $D_2 = [03, 05, 07, 09, 010]$ are shown as follows:

$$
\begin{align*}
&\mathcal{D}_{DS_{DIS_1}}(01) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(01) = 0, \quad \mathcal{D}_{DS_{DIS_3}}(01) = 0, \quad \mathcal{D}_{DS_{DIS_4}}(01) = 0, \quad \mathcal{D}_{DS_{DIS_1}}(02) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(02) = 0, \\
&\mathcal{D}_{DS_{DIS_3}}(04) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(04) = 0, \quad \mathcal{D}_{DS_{DIS_3}}(05) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(05) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(06) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(06) = 0, \quad \mathcal{D}_{DS_{DIS_3}}(07) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(07) = 0, \quad \mathcal{D}_{DS_{DIS_3}}(08) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(08) = 0, \quad \mathcal{D}_{DS_{DIS_3}}(09) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(09) = 0, \quad \mathcal{D}_{DS_{DIS_3}}(10) = 0, \quad \mathcal{D}_{DS_{DIS_2}}(10) = 0, \\
&\mathcal{D}_{DS_{DIS_3}}(11) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(11) = 1, \quad \mathcal{D}_{DS_{DIS_3}}(12) = 1, \quad \mathcal{D}_{DS_{DIS_2}}(12) = 1.
\end{align*}
$$

In MsDIS, let $\alpha = 0.75$, then fusion results are calculated by the aggregation operators:

$$
\begin{align*}
&\bigotimes_{i=1}^{4} DIS_i(D_1)_{0.75} = \{01, 06, 08\}, \quad \bigotimes_{i=1}^{4} DIS_i(D_1)_{0.25} = \{01, 02, 04, 06, 07, 08, 010\}, \\
&\bigotimes_{i=1}^{4} DIS_i(D_2)_{0.75} = \{03, 05, 09\}, \quad \bigotimes_{i=1}^{4} DIS_i(D_2)_{0.25} = \{02, 03, 04, 05, 07, 09, 010\}.
\end{align*}
$$

The fusion approximate accuracy and fusion roughness are shown as follows:

$$
\begin{align*}
\varrho_{MSD(U/D)(0.75, 0.25)} & = \frac{\sum_{D_i \in U/D} \bigotimes_{i=1}^{4} DIS_i(D_i)_{0.75}}{\sum_{D_i \in U/D} \bigotimes_{i=1}^{4} DIS_i(D_i)_{0.25}} = \frac{6}{14}, \\
\varrho_{MSD(U/D)(0.75, 0.25)} & = 1 - \varrho_{MSD(U/D)(0.75, 0.25)} = \frac{8}{14}.
\end{align*}
$$

In MsDIS, fusion results are calculated by the optimism aggregation operators:

$$
\begin{align*}
&\bigotimes_{i=1}^{4} DIS_i(D_1) = \{01, 02, 04, 06, 08\}, \quad \bigotimes_{i=1}^{4} DIS_i(D_1) = \{01, 02, 04, 06, 08, 010\}, \\
&\bigotimes_{i=1}^{4} DIS_i(D_2) = \{03, 05, 07, 09\}, \quad \bigotimes_{i=1}^{4} DIS_i(D_2) = \{03, 05, 07, 09, 010\}.
\end{align*}
$$
The fusion approximate accuracy and fusion roughness are shown as follows:

\[
Q_{MSD}(U/D)^O = \frac{\sum_{D_i \in U/D} (\bigotimes_{i=1}^4 DIS_i(D_j))}{\sum_{D_i \in U/D} (\bigotimes_{i=1}^4 DIS_i(D_j))} = \frac{9}{11}, \quad \rho_{MSD}(U/D)^O = 1 - Q_{MSD}(U/D)^O = \frac{2}{11}.
\]

In MsDIS, fusion results are calculated by the pessimistic aggregation operators:

\[
\begin{align*}
\bigotimes_{i=1}^4 DIS_1(D_1) &= \{a_6\}, \\
\bigotimes_{i=1}^4 DIS_1(D_2) &= \{a_1, a_2, a_4, a_5, a_7, a_8, a_9\}, \\
\bigotimes_{i=1}^4 DIS_2(D_1) &= \{a_3, a_5, a_9\}, \\
\bigotimes_{i=1}^4 DIS_2(D_2) &= \{a_1, a_2, a_3, a_4, a_5, a_7, a_8, a_9, a_10\}.
\end{align*}
\]

The fusion approximate accuracy and fusion roughness are shown as follows:

\[
Q_{MSD}(U/D)^P = \frac{\sum_{D_i \in U/D} (\bigotimes_{i=1}^4 DIS_i(D_j))}{\sum_{D_i \in U/D} (\bigotimes_{i=1}^4 DIS_i(D_j))} = \frac{1}{4}, \quad \rho_{MSD}(U/D)^P = 1 - Q_{MSD}(U/D)^P = \frac{3}{4}.
\]

Obviously, \(Q_{MSD}(U/D)^P \leq Q_{MSD}(U/D)_{(0.75,0.25)} \leq Q_{MSD}(U/D)^O\).

4. Experimental analysis

In the third part of this paper, we proposed three groups of aggregation operators in order to directly integrate MsDIS. In this section, we prove the effectiveness and feasibility of the aggregation operators through a series of experiments. Firstly, we propose an algorithm (Algorithm 1) for calculating the fusion approximation accuracy of MsDIS. Meanwhile, we design Algorithm 2 to calculate the approximate accuracy of mean fusion in order to verify the effectiveness of the aggregation operators. Second, we download 9 data sets on UCI (http://archive.ics.uci.edu/ml/datasets.html). The specific information of the data sets is shown in Table 2. The entire experiment was run on a private computer. The specific operating environment, including hardware and software, is shown in Table 3. Finally, we did two groups of comparative experiments.

In order to make readers understand the mean fusion method mentioned in Algorithm 2 better, next we introduce the steps of mean fusion method.

The method of mean fusion: Given a \(M_5\) \(DIS = \{DIS_1, DIS_2, \ldots, DIS_q\}\). Firstly, the MsDIS is integrated into single-source decision information system (SSDIS) through

\[
NewDIS(x_j, a_k) = \text{fix}\left(\frac{\sum_{i=1}^q DIS_i(x_j, a_k)}{q}\right),
\]

where \(NewDIS(x_j, a_k)\) represents the fused information and \(DIS_i(x_j, a_k)\) represents the initial information under the \(i\)th source. Then, the upper and lower approximation sets are found from SSDIS. Finally, the Approximation accuracy can be computed.

We analyze the time complexity of the two algorithms as shown below. In Algorithm 1, from step 2 to step 14, we calculate the sum of the decision support characteristic function and the sum of the decision related characteristic function of each object in each decision class, and the time complexity is \(O(m \times n \times q)\); from step 15 to step 25, we fuse the MsDIS through the aggregate operator constructed in section 3, and the time complexity is \(O(m \times n)\). In Algorithm 2, from step 2 to step 10, we perform mean fusion on MsDIS, and the time complexity is \(O(n \times p \times q)\); from step 11 to step 21, we compute the upper and lower approximations of each decision class, and the time complexity is \(O(m \times n)\).

It is well known that it is not easy to directly obtain a MsDIS from a machine learning database. Therefore, we use the method of adding white noise and random noise into the original data set to obtain the data required for the experiment. Next, we introduce the method of adding white noise into the original data. First, generate \(q\) numbers \((n_1, n_2, \ldots, n_q)\) that satisfy the normal distribution. Add white noise as follows:
Algorithm 1: Calculating the fusion approximation accuracy of MsDIS.

<table>
<thead>
<tr>
<th>Input</th>
<th>$(\alpha, \beta)$, $M_5$DIS = ${D1S_1, D1S_2, \ldots, D1S_5}$, $U/D = {D_1, D_2, \ldots, D_m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Output</td>
<td>The fusion approximation accuracy of $U/D$</td>
</tr>
</tbody>
</table>

1. begin
2. for $k = 1 : m$ do /* the $m$ is the number of decision classes */
3.  for $i = 1 : n$ do /* the $n$ is the number of objects */
4.    $DS(i) = 0; DR(i) = 0;$
5.    for $j = 1 : q$ do /* the $q$ is the number of information sources */
6.      if $|x_j|_{D1S_i} \subseteq D_k$ then
7.        $DS(i) = DS(i) + 1;$
8.      end
9.      if $|x_j|_{D1S_i} \cap D_k \neq \emptyset$ then
10.     $DR(i) = DR(i) + 1;$
11.    end
12.   end
13. end
14. for $k = 1 : m$ do /* the $m$ is the number of decision classes */
15.  for $i = 1 : n$ do
16.    $\bigoplus_{j=1}^q DIS_j(D_k)_\alpha \leftarrow \emptyset; \bigoplus_{j=1}^q DIS_j(D_k)_\beta \leftarrow \emptyset;$
17.    for $j = 1 : q$ do
18.      if $\frac{\bigoplus_{j=1}^q DIS_j(D_k)_\alpha}{n} \geq \alpha$ then
19.        $\bigoplus_{j=1}^q DIS_j(D_k)_\alpha = \bigoplus_{j=1}^q DIS_j(D_k)_\alpha \cup \{x_i\};$
20.      end
21.      if $\frac{\bigoplus_{j=1}^q DIS_j(D_k)_\beta}{n} > \beta$ then
22.        $\bigoplus_{j=1}^q DIS_j(D_k)_\beta = \bigoplus_{j=1}^q DIS_j(D_k)_\beta \cup \{x_i\};$
23.      end
24.    end
25. end
26. return $\phi_{MSD}(U/D)(\alpha, \beta) = \frac{\sum_{k=1}^m \bigoplus_{j=1}^q DIS_j(D_k)_\alpha}{\sum_{k=1}^m \bigoplus_{j=1}^q DIS_j(D_k)_\beta};$

$$IS_i(x, a) = \begin{cases} IS(x, a) + n_i, & \text{if } 0 \leq |n_i| \leq 1, \\ IS(x, a), & \text{otherwise}. \end{cases} \tag{24}$$

Similarly, the method of adding random noise is as follows:

$$IS_i(x, a) = \begin{cases} IS(x, a) + r_i, & \text{if } 0 \leq |r_i| \leq 1, \\ IS(x, a), & \text{otherwise}. \end{cases} \tag{25}$$

In the original information system, the value of object $x$ under attribute $a$ is denoted by us as $IS(x, a)$. After adding noise, the corresponding value in the $i$th information system is recorded as $IS_i(x, a)$. Then we randomly select 40% of the data in the original information system to add white noise, the remaining 20% add random noise, and the rest keep the information of the original system unchanged. In the experiment of this paper, we construct 10 information sources in each original data set by adding noise. Finally, we obtain a multi-source decision information system.

4.1. Comparison of fusion approximation accuracy and approximation accuracy after mean fusion

In this subsection, we compare pessimistic aggregation operators, optimistic aggregation operators and $(\alpha, \beta)$-aggregation operators with mean fusion by approximate accuracy. Fig. 3 shows the approximation accuracy of pessimistic fusion, optimistic fusion, mean fusion and $(\alpha, \beta)$-fusion. The following conclusions can be drawn from the experimental results.

- In each data set, the fusion effect of the optimistic aggregation operators is the best, and the fusion effect of the pessimistic aggregation operators is the worst. For the $(\alpha, \beta)$-aggregation operators, we can adjust the fusion result by adjusting the threshold $(\alpha, \beta)$, so for practical applications, the $(\alpha, \beta)$-aggregation operators is more suitable for life.
Algorithm 2: Calculating the approximate accuracy of MsDIS after mean fusion.

```
Input: MsDIS = \{DIS_1, DIS_2, \ldots, DIS_q\}, U/D = \{D_1, D_2, \ldots, D_m\}
Output: The fusion approximation accuracy of U/D

1 begin
2 for \(i = 1 : n\) do
3    for \(k = 1 : p\) do
4      Sum\((x_i, a_k)\) ← 0;
5      for \(j = 1 : q\) do
6        Sum\((x_i, a_k)\) ← Sum\((x_i, a_k)\) + DIS\(_j\)(\(x_i, a_k\));
7      end
8      NIS\((x_i, a_k)\) ← \(\frac{\text{Sum}(x_i, a_k)}{q}\);
9    end
10   end
11 end
12 for \(k = 1 : m\) do
13   NIS\((D_k)\) ← ∅; \(\overline{\text{NIS}}\((D_k)\)\) ← ∅;
14 end
15 for \(i = 1 : n\) do
16  if \([x_i]_{\text{NIS}} \subseteq D_k\) then
17    \(\overline{\text{NIS}}\((D_k)\) = \overline{\text{NIS}}\((D_k)\) ∪ \([x_i]\)\);
18  end
19  if \([x_i]_{\text{NIS}} \cap D_k \neq ∅\) then
20    \(\overline{\text{NIS}}\((D_k)\) = \overline{\text{NIS}}\((D_k)\) ∪ \([x_i]\)\);
21  end
22 end
23 return : \(\overline{\text{q}}_\text{NIS}(U/D) = \frac{\sum_{k=1}^{m} |\overline{\text{NIS}}(D_k)|}{\sum_{k=1}^{m} |\overline{\text{NIS}}(D_k)|}\);
24 end
```

Table 2
Specific information about the data sets.

<table>
<thead>
<tr>
<th>No.</th>
<th>Dataset name</th>
<th>Abbreviation</th>
<th>Objects</th>
<th>Attributes</th>
<th>Decision classes</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Hayes-Roth</td>
<td>HR</td>
<td>160</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>Liver Disorders</td>
<td>LD</td>
<td>345</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>Balance Scale</td>
<td>BS</td>
<td>625</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>Banknote authentication</td>
<td>BA</td>
<td>1372</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>Winequality-Red</td>
<td>WR</td>
<td>1599</td>
<td>11</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>Wireless Indoor Localization</td>
<td>WIL</td>
<td>2000</td>
<td>7</td>
<td>4</td>
</tr>
<tr>
<td>7</td>
<td>Abalone</td>
<td>A</td>
<td>4177</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>8</td>
<td>Winequality-White</td>
<td>WW</td>
<td>4898</td>
<td>11</td>
<td>7</td>
</tr>
<tr>
<td>9</td>
<td>Nursery</td>
<td>N</td>
<td>12960</td>
<td>8</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 3
Specific information about the operating environment.

<table>
<thead>
<tr>
<th>Name</th>
<th>Model</th>
<th>Parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPU</td>
<td>Intel(R) Core(TM) i3-2370M</td>
<td>2.40 GHz</td>
</tr>
<tr>
<td>Platform</td>
<td>MATLAB</td>
<td>R2016b</td>
</tr>
<tr>
<td>System</td>
<td>Windows 7</td>
<td>64 bit</td>
</tr>
<tr>
<td>Memory</td>
<td>DDR3</td>
<td>4 GB;1600 Mhz</td>
</tr>
<tr>
<td>Hard Disk</td>
<td>MQ01ABD050</td>
<td>500 GB</td>
</tr>
</tbody>
</table>

Moreover, as shown in Fig. 3, in each data set, we can find such a set of thresholds that its fusion effect is better than the mean fusion effect.

- In the data sets named “Hayes-roth”, “Balance Scale”, “Banknote authentication” and “Nursery”, when the values of the threshold \(\alpha\) are taken to 0.6, 0.7, 0.8, and 0.6, respectively, the fusion effect is still better than that of the mean fusion. This shows that the gap between the different information sources in these four data sets is not particularly large, and the decision results are relatively consistent. Conversely, in the data sets named “Liver Disorders”, “Winequality-Red”, “Wireless Indoor Localization”, “Abalone” and “Winequality-White”, threshold \(\alpha\) should not exceed 0.3, 0.3, 0.2, 0.2 and 0.3 respectively if the fusion effect is better than that of mean fusion. This means that the gaps between the various sources in these five data sets are large, and the decision results are quite different.
4.2. Comparison of fusion approximation accuracy under different thresholds

We set three groups thresholds for comparison with pessimistic and optimistic. The three groups thresholds are (0.4,0.6), (0.5,0.5) and (0.6,0.4), respectively. We calculate the fusion approximation accuracy under these three groups thresholds, and then compare it with the pessimistic fusion approximation accuracy and the optimistic fusion approximation accuracy. It can be seen from the Fig. 4: the fusion approximation accuracy of the optimistic aggregation operators is the largest, and the fusion approximation accuracy the pessimistic aggregation operators is the smallest. The fusion approximation accuracy of the $(\alpha, \beta)$-aggregation operators is between the pessimistic approximation accuracy and the optimistic approximation accuracy, and decreases with the increase of threshold $\alpha$. Obviously, the experimental results satisfy the Property 3.5.

5. Conclusions

In this paper, a novel fusion method of MsDIS is proposed from the perspective of multi-granulation. Firstly, the decision support characteristic function and the decision related characteristic function are constructed. Secondly, aggregation operators (including pessimistic aggregation operators and optimistic aggregation operators) induced by two functions are defined and the related properties of aggregation operators are researched. Meanwhile, for different aggregation operators, different fusion approximation precision is defined. These different fusion approximation precision provide measurement for our experiments. Finally, it is proved by experiments that the fusion method proposed by us has practicality when the gap between the sources is small. In the future, we will further study the fusion methods of heterogeneous multi-source information systems.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
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