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Regret-theory-based three-way decision making in hesitant fuzzy environments: A multi-attribute approach and its applications

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Abstract
Decision-making is intricately linked to the psychological behavior of decision-makers, particularly their susceptibility to risk uncertainty and the consequent emergence of regret psychology. The hesitant fuzzy information system is an effective mechanism for encapsulating the substantial uncertainty inherent in real-world data. While existing three-way multi-attribute decision-making (TWD-MADM) methods have made significant progress in handling uncertainty, they often overlook the psychological factors of decision-makers, such as regret aversion. This paper introduces a three-way decision-making method (TWD-MADM-RT-HFS), grounded in regret theory, for multi-attribute decision-making in a hesitant fuzzy environment. Unlike traditional TWD-MADM approaches, our method explicitly incorporates regret theory to model decision-makers' psychological behavior, providing a more realistic framework for decision-making under uncertainty. The methodology involves computing a relative outcome matrix using the PROMETHEE-II method to assess the gains and losses of objectives. A novel regret-based perceived utility function is proposed to quantify decision-makers' aversion to regret, followed by calculating satisfaction-based weight functions for different events across various states. The integration of these weight functions with the perceived utility function yields a new expected utility function, pivotal for ranking and classifying alternatives. To validate the effectiveness of the proposed methodology, the Algerian Forest Fires Dataset was selected for application testing and successfully classified into three categories: fire, possible fire and no fire. The results were then ranked in detail based on the probability of their occurrence. It is anticipated that this classification will help to predict fire risk more accurately in the future, so that timely measures can be taken to prevent and control fire hazards. The method's feasibility, effectiveness, and superiority are validated through a comparative analysis with existing methods in real-case scenarios. The stability of the model is further confirmed by conducting sensitivity analyses under different parameter settings.

Keywords Three-way decision · Multi-attribute decision making · Regret theory · Hesitant Fuzzy environment · Perceived utility function

1 Highlights

- Research highlight 1: The objective of this paper is to propose a new perceived utility function that is always non-negative. The interpretability and credibility of the model can be improved, and it also helps decision makers to better understand the outputs of the model.
- Research highlight 2: This paper presents a novel three-way decision model based on regret theory, developed in a hesitant fuzzy environment for the first time, which can improve the efficiency and accuracy of decision-making.
- Research highlight 3: The proposed model takes into account the psychological behavior of the decision maker, and instead of trying to minimize risk, the expected utility function pairs are used to establish ranking and classification principles.
- Research highlight 4: By weighting events from a satisfaction perspective, the importance and impact of

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different events can be assessed more objectively, improving the accuracy and credibility of the model.

- Research highlight 5: The proposed model's has both ranking and classification with good classification performance and low CER.

2 Introduction

The consequences of forest fires for the environment and ecosystems are profound, resulting in the loss of biodiversity, destruction of vegetation and animal habitats, and disturbances to ecological balance. Furthermore, forest fires release toxic gases and particulate matter, which contribute to air pollution, pose a risk to human health, and exacerbate the effects of global climate change. Forest fires can cause soil erosion, desertification, and degradation, as well as negatively impacting water resources. The destruction of protected areas, flash floods, and a decline in water quality directly threaten the security of water supplies. Forest fires have a significant economic impact, including the burning of trees and crops and the destruction of infrastructure, resulting in heavy economic losses to local communities and affecting people's livelihoods and ways of making a living. Therefore, it is essential to prioritize forest fire prevention and strengthen forest protection to safeguard natural ecosystems and ensure the sustainable development of human societies. This paper verifies the effectiveness of the proposed model in solving a real-world decision-making problem by using a realistic forest fire case study, Algerian Forest Fires Dataset (https://archive.ics.uci.edu).

2.1 A brief review of three-way decision for multi-attribute decision making

Multi-attribute decision making (MADM) is the process of making a decision when faced with multiple attributes or criteria in a complex and changing environment. Decision making in modern society often involves multiple interests, uncertainties, and risks, and a single attribute cannot fully characterize a problem in its entirety. As a result, multi-attribute decision making has become a common approach to decision making. In such a context, decision makers need to comprehensively consider multiple attributes in order to balance the needs of all parties and make the best decision.

Based on the rough set theory proposed by [26, 48] quantified the losses and decision costs of the objects in different states to establish the three-way decision (TWD) model. This decision-making method uses three different regions in the field of rough set theory to approximate various concepts with rich semantic interpretation. Specifically, TWD divides the decision space into three regions: *positive region* corresponds to the decision that decision makers are willing to accept, i.e., the decision that meets the requirements or expectations; *boundary region* corresponds to the region that indicates that the decision maker needs to further consider or delay the decision because these decisions may oscillate between acceptance and rejection and need more information or evaluation; *negative region* corresponds to the decision that the decision maker does not want to accept, i.e., the decision that does not meet the requirements or expectations. In TWD, the decision maker must divide the decision space into these three regions according to the intricacy of the issue and reliable data, and make the decision to accept, delay, or reject accordingly. This approach helps the decision maker better understand the uncertainty and complexity of the problem and make more informed decisions.

Over the past decade, TWD has seen significant advancements in theory, methodology, and applications. Yao's foundational work [48] established the theoretical basis for TWD by integrating decision-theoretic rough sets (DTRS) and introducing loss functions to evaluate the costs of different decision actions. This framework has been further extended and refined by subsequent researchers. For example, [6] integrated classical TWD models into a broader mathematical framework, exploring their applications to spatial decision-making problems. [47] also combined TWD with cognitive computing, providing a new research paradigm that bridges decision-making and human cognition. Furthermore, Ciucci and Dubois [4] investigated the three-valued logic underlying TWD, offering a formal basis for its application in logical reasoning and uncertainty analysis. Recently, [45] proposed a novel framework of sequential three-way decision for the fusion of mixed data from subjective and objective dynamic perspectives, emphasizing the importance of integrating dynamic data and decision-making behavior. Their work provides a valuable reference for extending our model to sequential decision-making scenarios.

In terms of methodology, several innovative models and algorithms have been proposed to enhance the applicability and effectiveness of TWD. [52] developed a TWD model based on utility theory, incorporating decision makers' preferences and risk attitudes into the decision-making process. [3] proposed a region-preserving approximation method for neighborhood systems using conditional entropy and heuristic algorithms, which improves the accuracy of the decision region in complex environments. These methods have been applied in various fields, such as cost-sensitive portrait recognition [12], spam filtering [55], and uncertainty analysis in medical decision-making [46]. Recently, a novel three-way decision model integrated with fuzzy sets was proposed by [13], which effectively handles uncertainty and complex decision-making scenarios by combining fuzzy set theory with the three-way decision framework.

The scope of TWD has also expanded beyond traditional decision-making contexts. Concepts such as TWD thinking [6, 44], TWD methodology [50], and TWD processing [43] have emerged, reflecting the growing recognition of TWD as a versatile framework for addressing complex and uncertain problems. These developments have enriched the theoretical foundations of TWD and provided new tools for multi-attribute decision-making (MADM). For instance, TWD has been employed in the context of MADM problems by integrating attribute weights, loss functions, and decision thresholds, thereby empowering decision makers to evaluate alternatives in a more systematic and robust manner. Based on the advantages of the three-way decision model, in this paper we aim to develop a new TWD model that is more adaptable to uncertain environments and more reliable in practice.

2.2 A brief review of hesitant fuzzy systems

In 2010, [31] organically combined fuzzy set theory and hesitant theory to introduce the concept of Hesitant Fuzzy Systems (HFSs), which provides an efficient solution for dealing with uncertainty and ambiguity in the real world. The work of Xia and Xu [40] provided an important mathematical expression for high-frequency systems, an expression that became the cornerstone of subsequent research. Their contributions greatly advanced the development of the theory of high-frequency systems. Subsequently, the development of HFS theory has shown several branches with the depth of research. In addition, the theory was extended not only in theoretical studies but also in practice. For example, [41] applied it to distance metrics, and [18] applied it to subtraction and division operations.

In recent years, with the in-depth study of hesitant fuzzy environments, many methods have been born. In the context of hesitant fuzzy environments, TOPSIS [42] utilises the calculation of distances between fuzzy alternatives and an ideal solution, thereby facilitating rapid and efficient decision-making. This method is particularly beneficial in scenarios where decision criteria are inherently uncertain, such as in environmental impact assessments or healthcare resource allocation. [38] proposed a novel hesitant fuzzy decision-making approach based on improved TOPSIS, which is specifically applied to risk assessment in hesitant fuzzy environments; Conversely, TODIM [54] integrates hesitant fuzzy sets to reflect the nuanced preferences of decision makers, thereby providing a deeper understanding of their cognitive processes in the face of ambiguity. This approach is well-suited for complex decision contexts, such as multi-criteria product selection or project evaluation, where subjective judgments play a significant role; Additionally, [49] introduced a novel three-way decision model based on α-fuzzy neighborhood classes, which effectively handles uncertain data in multi-criteria decision-making problems, further enriching the theoretical foundation of decision-making in hesitant fuzzy environments. The ARAS method [24] developed by Mishra et al. provides an effective solution for decision analysis and optimization in this environment. By taking into account the special factors in HF environment, the ARAS method is able to better adapt to uncertainty and risk, and provide decision makers with reliable decision support to help them make the best choices in complex decision environments; [11] presented a rough set model based on the decision theory of fuzzy binary relations, which can effectively deal with fuzzy uncertainty problems; Liang and Liu's model [16] introduces the hesitant fuzzy loss function into the processing, which improves the credibility and accuracy in complex situations; [35] proposed a multi-attribute TWD model based on prospect theory, which takes into account people's subjective perception of risk. Combining regret theory and prospect theory, [14] proposed a multi-attribute decision-making method based on hesitation fuzzy sets, which takes into account the effects of regret and risk perception on the decision-making process, and further enhances the adaptability and reasonableness of the decision-making model in uncertain environments. [37] proposed a multi-attribute decision-making method based on hesitant fuzzy sets and regret theory, which effectively captures the psychological behaviors of decision-makers, such as regret aversion and risk perception.

Under the HF-TWD framework, there have been methods [35] to study the impact of decision outcomes based on prospect theory [33]. In view of this, this paper chooses to propose a new model based on regret theory to provide better theoretical support to solve practical problems.

2.3 A brief review of regret theory

Regret Theory (RT) represents a significant branch of decision theory that endeavors to elucidate the potential for individuals to experience regret during the decision-making process and to examine the impact of such experiences on their subsequent choices.

Regret Theory, proposed by [1] in 1982, is an important perspective in decision theory. The theory focuses on an individual's regret for different decision outcomes and incorporates this emotion into the consideration of the decision process. Regret theory suggests that when people make decisions, they consider not only the expected utility, but also the regret that may result. If the outcome of a decision is far from the individual's expectations, resulting in a greater degree of regret, then the individual may make different decision choices to avoid or mitigate the degree of regret.

In recent years, as regret theory has been studied in depth, scholars have also begun to explore its applicability in more complex contexts, such as its role in group decision making [29] and integrated decision making [53]. In this regard, [30] conducted a study on a group decision-making approach that is based on a regret theory, which integrates probabilistic linguistic term sets with a view to better handling uncertainty and imprecision in group decisions.[7] define the dominance relationship based on the regret-elation value of each option under each attribute and then design the relative utility function using the relative loss function. Moreover, [8] proposed a regret theory-based three-way decision approach for multi-scale decision information systems, enhancing the model's applicability in uncertain environments.

Taken together, the research on regret theory has been deepened and expanded, providing an important theoretical foundation for a better understanding of human decision-making behavior, and is of great significance as a guide for practical application. And there are relatively few studies on regret theory in decision making in hesitation fuzzy environment, so this paper establishes a novel model on the basis of regret theory in hesitation fuzzy environment.

2.4 Rationale underpinning and pivotal innovations

Based on the research results of the above theories, the main objective of this paper is to synthesize the theories and methods of HFSs, RT, TWD, the PROMETHEE-II, and establish a new three-way decision method of multi-attribute decision making based on regret theory in hesitant fuzzy environment (TWD-MADM-RT-HFS). We delineate the rationale underpinning the TWD-MADM-RT-HFS approach as detailed below.

(1) Regret theory, while valuable, lacks an inherent classification mechanism a shortcoming particularly evident in high-object HF-MADM scenarios where decision-makers prefer models offering both classification and ranking capabilities. This paper introduces the TWD-MADM-RT-HFS model, designed for rational and efficacious ranking and classification.
(2) Although existing studies have proposed the MADM based on the 3WD method with fuzzy hesitant information [34], the integration of regret theory within contexts of ambiguous hesitant decision-making has been relatively unknown. To address this gap, we propose a novel model that applies regret theory to such environments.
(3) In the traditional TWD framework, the loss function and state set are often subject to individual interpretation, leading to variability in the model's construction and application. This paper presents the TWD-MADM-RT-HFS method, which aims to derive utility functions and event weights in an objective manner.
(4) While [51] have demonstrated the model's validity through investment project case studies, the limited number of objects may undermine the model's stability and reliability. To counter this, we have substantiated the robustness of our proposed model through an extensive case study involving 244 objects.

This manuscript introduces several pivotal innovations as follows:

(1) We introduce a novel perceived utility function, inherently non-negative, which enhances the model's interpretability and credibility, thereby facilitating a clearer understanding of the model's outcomes for decision-makers.
(2) We pioneer the development of a three-way decision model grounded in regret theory within a hesitant fuzzy context, offering significant advancements for addressing intricate real-world decision-making scenarios and enhancing decision-making efficiency and precision.
(3) Our model integrates both ranking and classification capabilities, accounting for the psychological aspects of decision-making. It goes beyond traditional risk minimization by using pairs of expected utility functions to formulate ranking and classification criteria, not only improving the accuracy of decision-making but also providing a more nuanced understanding of risk by combining the strengths of categorization and sorting.
(4) By employing a satisfaction-based weighting of events, our approach provides a more objective assessment of the significance and impact of various events, thereby bolstering the model's accuracy and reliability. This methodology diminishes the model's subjectivity, ensuring it more closely aligns with the actual requirements and preferences of stakeholders.

2.5 Structural framework

Figure 1 depicts the principal structural framework of this paper. The rest of the article is structured as follows: In Section 2, we review the HFSs, RT, TWD, the basic concepts of PROMETHEE-II. In Section 3, we define a new perceptual utility function; a new TWD-MADM-RT-HFS

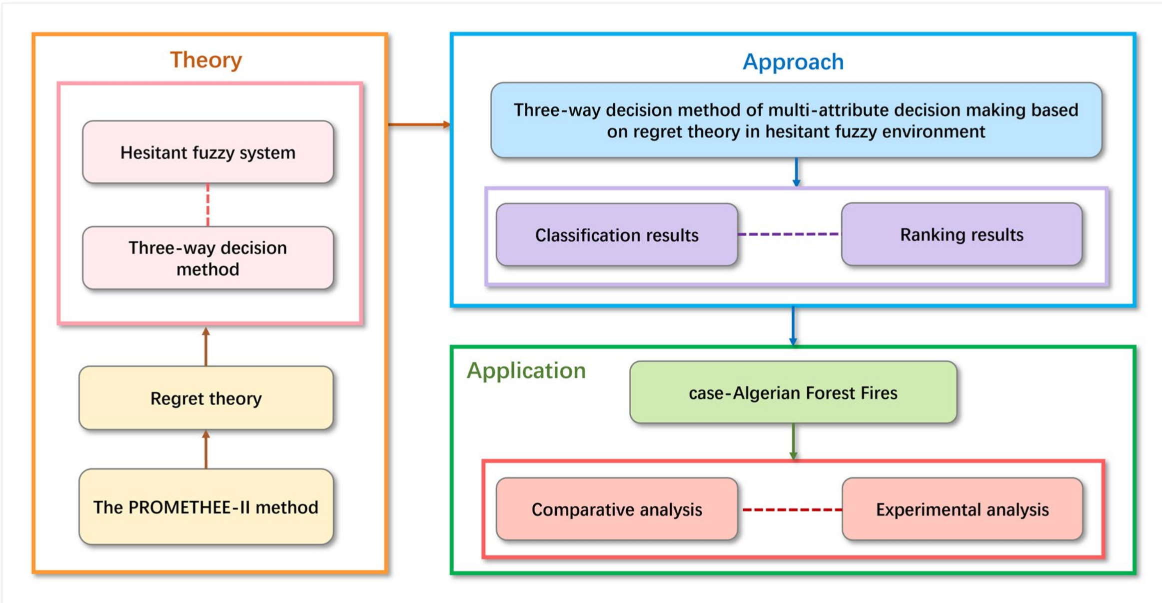

Fig. 1 The structural framework of this paper

method by combining RT with TWD in the HF environment. In Section 4, we solve practical problems with the proposed TWD-MADM-RT-HFS and analyze the proposed model in comparison with existing methods to illustrate the rationality and validity of the proposed model. In Section 5, we discuss the effect of parameter variations on the results. In Section 6, we summarize some of the main research elements and possible directions for future research.

3 Preliminaries

To enhance the readability of this paper, we begin by defining the common symbols used throughout, as summarized in Table 1.

3.1 HFSs

As there are several possible values for the affiliation of each element, [31, 32] proposes the definition of HFS for this case. Let V is a fixed set and the hesitant fuzzy set (HFS) on V is a function by which V can be mapped to the set $[0, 1]$. In order to facilitate comprehension, [40] provide a mathematical representation of HFS.

Given a reference set V, the expression for the HFS on V is

$$E = \{\langle v, \quad h_E(v)\rangle \mid v \in V\},$$

where $h_E(v)$ is the possible degree of affiliation of $v \in V$ belonging to E, i.e. $h_E(v)$ is the set of some numbers on $[0, 1]$. Thus, we define $h_E(v)$ as a hesitant fuzzy element (HFE) and V is the set of all hesitant fuzzy elements.

Next, [40] defines some operations for HFE. Due to the high computational complexity, [19] et al. modified these operations by defining the following operations:

Table 1 The definitions of the common symbols

Symbol	Definition
v_{ik}	The evaluated value of object s_i for attribute C_k
$v_i^{\psi(l)}$	The lth value in the ascending order of v_i
w_k	The weight of the attribute of kth
C_k	The kth conditional attribute
D	The HF deviation
P	The hesitant preference degree
U	The direct utility function
R	The regret function
MU	The regret-based perceived utility function
θ	The risk aversion parameter
δ	The regret aversion parameter
ρ	The dominance degree
$\mathbb{A}$	Action (acceptance, observation, and rejection respectively)
$\mathbb{S}$	State or event
π	The global preference value
π^-	The overall loss value
π^+	The overall gain value

Suppose $v_i\,(i,j=1,2,\ldots,m)$ are hesitant fuzzy elements, and the number of elements of both v_i and v_j is n, then

$$v_i^a=\left\{\left(v^{\psi(l)}\right)^a,l=1,2,\ldots,n\right\};$$
$$av_i=\left\{1-\left(1-v^{\psi(l)}\right)^a,l=1,2,\ldots,n\right\};$$
$$v_i\oplus v_j=\left\{v_i^{\psi(l)}+v_j^{\psi(l)}-v_i^{\psi(l)}v_j^{\psi(l)},l=1,2,\ldots,n\right\};$$
$$v_i\otimes v_j=\left\{v_i^{\psi(l)}v_j^{\psi(l)},l=1,2,\ldots,n\right\};$$
$$\oplus_{i=1}^m v_i=\left\{1-\prod_{i=1}^t\left(1-v_i^{\psi(l)}\right),l=1,2,\ldots,n\right\};$$
$$\otimes_{i=1}^m v_i=\left\{\prod_{i=1}^t v_i^{\psi(l)},l=1,2,\ldots,n\right\},$$

where a is a positive real number, $v_i^{\psi(l)}$ is the lth value in the ascending order of v_i.

In practice, the base equivalence condition in the definition of hesitant fuzzy sets usually cannot be satisfied. Therefore, [41] propose a solution that aims to satisfy the base equivalence condition and better adapt to practical needs by adding elements to shorter hesitant fuzzy sets according to the decision maker's risk preferences.

Specifically, when the number of elements in one hesitant fuzzy set is greater than another set, different operating rules are applied depending on whether the decision maker is an optimist or a pessimist. For optimists (pessimists), the maximum (minimum) value is taken from the smaller set and added to the smaller set until the number of elements in the two sets is equal.

The strength of this approach is that it takes into account the risk preference of the decision maker, which makes the hesitant fuzzy set more applicable to practical problems.

3.2 RT

Suppose that an investor must must choose between investment options A and B. Option A is a sound investment with relatively stable expected returns, while Option B is a riskier choice that may bring higher returns but also comes with higher risks. If Option A is chosen and stable returns are achieved, there may be little regret as the outcome aligns with the decision. However, if Option A is chosen and it is discovered that Option B would have yielded greater returns, there may be regret for missing out on a better opportunity. Conversely, if Option B is chosen and a significant loss is suffered, there may be regret for choosing a riskier option.

[1] proposed a theory of regret which sees regret as an emotion that arises as a result of comparing the outcome or state of affairs of a particular event. This example vividly illustrates the concept of regret theory. Individuals often avoid decisions that may result in significant feelings of regret, opting instead for relatively safe options. Regret can significantly impact decisions related to investments and purchases, influencing our behaviour and preferences. A deeper understanding of regret theory can provide clarity on our decision-making tendencies, enabling us to make more informed choices when faced with a decision.

Bell defines the modified or collective utility function for alternative p as:

$$MU(p,q)=U(p)+R(U(p)-U(q)),\tag{1}$$

where $U(p)$ is a value function of the direct utility of adopting alternative p, and $R(U(p)-U(q))$ is an indirect utility function that measures the value of regret between p and q.

In this instance, $\triangle U=U(p)-U(q)$. In the outcome obtained by [2], the direct utility function and the regret function are defined as respectively:

$$U(p)=\frac{1-e^{-\theta\cdot p}}{\theta},\tag{2}$$

$$R(\triangle U)=1-e^{-\delta\cdot\Delta U},\tag{3}$$

where $\theta\in[0,+\infty)$ is the risk aversion parameter and $\delta\in[0,+\infty)$ is the regret aversion parameter.

[27] extends the application of regret theory to multiple action sets and solves real-world decision problems involving multiple choices. He proposes a more general formula for selecting goal-maximising options. Assuming the outcomes of alternatives A_1, A_2, ... ,A_i are a_1, a_2, ... ,a_i, the perceived utility function of regret for alternative A_i is as follows:

$$MU(A_i)=U(A_i)+R(U(A_i)-U(A_\star)),\tag{4}$$

where $A_\star=\max\{A_i\mid i=1,2,\ldots,m\}$. $R(U(A_i)-U(A_\star))$ denotes the regret value and is non-positive.

3.3 TWD

Yao ([47]) introduced the narrow three-way decision theory (TWD) within the context of a probabilistic rough set model, which is based on a Bayesian decision-making process. This was achieved by analysing the decision rules of classical rough sets and decision-theoretic rough sets.

In the past few years, researchers have developed the three-way decision making model through subject-object dynamic fusion. This model brings new ideas and methods for decision making in multi-criteria environments. The TWD model combines the loss function and evaluation value to comprehensively consider multiple factors in the decision-making process. Its development into the application of relative loss functions [9] allows decision makers to assess various possible consequences more accurately and effectively. It increases flexibility and provides support and guidance for solving complex problems by dynamically integrating subject and object.

The TWD model expands the scope of decision-making theory and provides an innovative solution to practical decision-making problems. The TWD model enhances the decision-making process by comprehensively considering multiple factors, providing decision makers with more support and choice paths. It opens up new possibilities for research and practical application in contemporary decision-making theory, injecting more wisdom and scientific rigour into the process.

Assume that U is a non-empty, finite set. $\mathbb{A} = \{A_p, A_b, A_n\}$ is a set of actions and A_p, A_b, and A_n in a state denotes acceptance, observation, and rejection respectively. $\mathbb{S} = \{S, \neg S\}$ is a set of states and S implies that the element x belongs to decision class S, while $\neg S$ implies that it does not. v_{ik} represents the evaluated value of object s_i for attribute C_k. v_k^{max} and v_k^{min} denote the largest evaluated attribute and the smallest evaluated attribute under attribute C_k. The argument domain U is split into three separate non-overlapping fields: the positive region (*Pos*(S)), the negative region (*Neg*(S)) and the boundary region (*Bnd*(S)), where $U = Pos(S) \cup Bnd(S) \cup Neg(S)$ and $\emptyset = Pos(S) \cap Bnd(S) \cap Neg(S)$. Different actions trigger different risks in the two states, let θ be the risk aversion coefficient, then the relative loss function is shown in Table 2.

3.4 The PROMETHEE-II method

In 1985, [22] proposed the Preference Ranking Organisation METHod for Enrichment Evaluations II (PROMETHEE-II). [25] extended the PROMETHEE method to hesitant fuzzy environments, offering a more comprehensive decision-making tool for complex scenarios. [21] applied a PROMETHEE-based approach to sustainable supplier selection under hesitant fuzzy information, demonstrating its practical utility in real-world decision-making.

In PROMETHEE-II, the relative outcome function is determined by defining a preference function to quantify the decision maker's preference for different criteria values and constructing a relational judgment matrix to compare the order of merit of different solutions. The performance of each solution is evaluated by calculating the outflow and inflow flows under each criterion. A relative outcome function is then defined to combine the performance under each criterion and quantify the overall performance of each solution. This approach aids decision-makers in comprehending the relationship between solution strengths and weaknesses, supports multi-criteria decision analysis, and enables clear comparison and evaluation of different solutions.

Table 2 The relative loss function

	S_k	$\neg S_k$
A_p	0	$v_k^{max} - v_{ik}$
A_b	$\theta(v_{ik} - v_k^{min})$	$\theta(v_k^{max} - v_{ik})$
A_n	$v_{ik} - v_k^{min}$	0

4 TWD method based on RT in HF environment

4.1 Calculation on the regret-based perceived utility function in HF environment

4.1.1 The relative outcome function in HF environment

The HF-PROMETHEE-II methodology and the PROMETH EE-II methodology share similarities in measuring object gains and losses. The HF-PROMETHEE-II method uses the leaving stream to measure an object's advantage relative to others, and the entering stream to measure its disadvantage by calculating the gap between the object and others. This allows for estimation of the outcome value by assessing the degree of advantage or disadvantage under each criterion. This method combines the concepts of HF-PROMETHEE-II and the PROMETHEE-II framework to provide a more comprehensive analysis of multi-criteria decision making, thereby improving the accuracy and reliability of decision-making.

In the HF environment, suppose there are m attributes and n objects that satisfy $\sum_{k=1}^{m} w_k = 1$, where w_k is the weight of the attribute of kth, v_{ik} is the evaluation value of the ith object to the kth attribute.

To calculate the relative outcome matrix in a hesitation ambiguity setting, follow these main steps:

First, the hesitant fuzzy deviation [18] under the attribute C_k is used to determine the gains and losses of the object s_i with respect to other objects. The HF deviation is given by Equation 5.

$$D_k(s_i, s_j) = v_{ic} \ominus v_{jc} = \cup_{\varphi_1 \in v_{ik}, \varphi_2 \in v_{jk}} \{\varpi\}, \tag{5}$$

where

$$\varpi = \begin{cases} \frac{\varphi_1^{\psi(l)} - \varphi_2^{\psi(l)}}{1 - \varphi_2^{\psi(l)}}, & \text{if } \varphi_1^{\psi(l)} \geq \varphi_2^{\psi(l)} \text{ and } \varphi_2^{\psi(l)} \neq 1, \\ 0, & \text{otherwise}, \end{cases} \tag{6}$$

where $\varphi_i^{\psi(l)}$ is the lth number in the ascending order of φ_i.

Table 3 The relative outcome function in HF environment

	S_k	$\neg S_k$
A_p	$\pi^+(s_i)$	0
A_b	$\xi \cdot \pi^+(s_i)$	$\xi \cdot \pi^-(s_i)$
A_n	0	$\pi^-(s_i)$

Secondly, to effectively utilize the PROMETHEE-II method, we first define a preference function for each criterion to quantify the decision maker's preference for different attribute values. In this study, we employ a Gaussian preference function to calculate the degree of hesitant preference under each attribute. In Gaussian theory, ρ represents the curvature of Gaussian. The larger the value of ρ, the smaller the curve curvature. In preference function theory, ρ represents the degree of dominance. Based on the majority rule principle, the ρ value in this model takes the range of $[0.5, 1]$. Therefore, the method of calculating degree of hesitant preference under a single attribute is given by Equation 7.

$$P_k(s_i, s_j) = 1 - e^{-\frac{(D_k(s_i,s_j))^2}{2\rho_k^2}}. \tag{7}$$

In practical problems, an event in a hesitant fuzzy environment often has several attributes, each of which has a different importance or weight. It may then be considered to aggregate the hesitation preferences of individual attributes according to the attribute weights, in order to consider the influence of each attribute on the final decision in a more comprehensive and integrated way.

In this paper, we use the hesitant fuzzy weighted average (HFWA) operator defined by [40] to aggregate information about different attributes, from which we obtain the global preference value of the object.

$$\text{HFWA}(s_1, s_2, \ldots, s_n) = \oplus_{k=1}^{n}(w_k s_k),$$

$$\text{HFWA}(s_1, s_2, ..., s_n) = \cup_{\varphi_1 \in s_1, \varphi_2 \in s_2, ..., \varphi_n \in s_n}\{1 - \textstyle\prod_{k=1}^{n}(1-\varphi_k)^{w_k}\}$$

where $w = (w_1, w_2, \ldots, w_n)^T$ is the weight vector of $s_k = (1, 2, ..., n)$ with $w_k \in [0, 1]$ and $\sum_{k=1}^{n} w_k = 1$.

The global preference value for an object is given by Equation 8.

$$\pi(s_i, s_k) = \oplus_{k=1}^{n}(w_k P_k(s_i, s_j)). \tag{8}$$

For the case where the outcome value of an object is divided into gains and losses, in order to measure the overall loss value of object s_i relative to other objects, the following formula can be used for the calculation:

$$\pi^-(s_i) = \oplus_{j=1}^{m}\pi(s_j, s_i). \tag{9}$$

The total gain value of object s_i in relation to the other objects is given in Equation 10.

$$\pi^+(s_i) = \oplus_{j=1}^{m}\pi(s_i, s_j). \tag{10}$$

After combining the elements of regret theory with the overall gain and loss values in the HF environment, we constructed the relative outcome matrix for object s_i in the HF environment as shown in Table 3, where ξ is the utility pursuit coefficient [56] used to represent the outcome of an object.

As there are too many objects in the forest fire dataset to present as an example, this paper will demonstrate the calculation process using an arithmetic example [40] with only four objects.

Example 1.1 The board of directors of a company needs to evaluate four possible projects $\mathbb{S} = \{s_1, s_2, s_3, s_4\}$, taking into account four maximising attributes $\mathbb{C} = \{C_1, C_2, C_3, C_4\}$ such as finance, customer satisfaction, internal business processes and learning and growth. The weights of each attribute are shown in the vector $w = (0.2, 0.3, 0.15, 0.35)^T$. Need to select the best project and sort all the projects. Assuming a degree of dominance of $\rho = 0.8$ and a utility coefficient of $\xi = 0.5$.

The decision matrix $V = (v_{ik})_{nm}$ is shown in Table 4, where v_{ik} is in HFE form.

The first step is to calculate the HF deviation of object s_1 with respect to object s_2 under attribute C_3.

$$\begin{aligned} D_3(s_1, s_2) &= v_{13} \ominus v_{23} \\ &= \{0.2, 0.3, 0.6, 0.7, 0.9\} \ominus \{0.3, 0.3, 0.4, 0.6, 0.9\} \\ &= \{0, 0, 0.3333, 0.25, 0\}. \end{aligned}$$

The second step is to calculate the degree of hesitation preference of s_1 with respect to s_2 under C_3.

Table 4 The hesitant fuzzy decision matrix

	C_1	C_2	C_3	C_4
s_1	$\{0.2, 0.4, 0.7\}$	$\{0.2, 0.6, 0.8\}$	$\{0.2, 0.3, 0.6, 0.7, 0.9\}$	$\{0.3, 0.4, 0.5, 0.7, 0.8\}$
s_2	$\{0.2, 0.4, 0.7, 0.9\}$	$\{0.1, 0.2, 0.4, 0.5\}$	$\{0.3, 0.4, 0.6, 0.9\}$	$\{0.5, 0.6, 0.8, 0.9\}$
s_3	$\{0.3, 0.5, 0.6, 0.7\}$	$\{0.2, 0.4, 0.5, 0.6\}$	$\{0.3, 0.5, 0.7, 0.8\}$	$\{0.2, 0.5, 0.6, 0.7\}$
s_4	$\{0.3, 0.5, 0.6\}$	$\{0.2, 0.4\}$	$\{0.5, 0.6, 0.7\}$	$\{0.8, 0.9\}$

$$P_3(s_1, s_2) = \{1 - \mathrm{e}^{-\frac{0^2}{2*0.8^2}}, 1 - \mathrm{e}^{-\frac{0^2}{2*0.8^2}}, 1 - \mathrm{e}^{-\frac{0.3333^2}{2*0.8^2}}, 1 - \mathrm{e}^{-\frac{0.25^2}{2*0.8^2}}, 1 - \mathrm{e}^{-\frac{0^2}{2*0.8^2}}\}$$
$$= \{0, 0, 0.0831, 0.0477, 0\}.$$

Similarly, the hesitation preference of object s_1 with respect to object s_2 under the other attributes is computed:

$$P_1(s_1, s_2) = \{0\};$$
$$P_2(s_1, s_2) = \{0, 0.0096, 0.0831, 0.2452\};$$
$$P_4(s_1, s_2) = \{0\}.$$

The third step is to aggregate the information about the different attributes to calculate the global preference value of s_1 in relation to s_2.

$$\pi(s_1, s_2) = (w_1 P_1(s_1, s_2)) \oplus (w_2 P_2(s_1, s_2)) \oplus (w_3 P_3(s_1, s_2)) \oplus (w_4 P_4(s_1, s_2))$$
$$= \{0, 0.0029, 0.0328, 0.0928\}.$$

Similarly, the global preference values of each attribute with respect to other attributes can be calculated by the Equation 8,the results are shown in the Table 5.

The fourth step is to calculate the overall profit and loss value, as shown in the Table 6. The fifth step is to compute the relative outcome function for object s_i, as shown in the Table 7.

4.1.2 Selections of the reference point

RT is a decision theory. [27] extends to situations involving multiple options, with the aim of helping decision-makers make optimal choices among multiple options. The decision maker needs to choose one of the action sets in order to maximise some goal.

To make the decision, the decision maker first evaluates the utility of each option, where utility refers to the value or satisfaction the decision maker places on each option. The decision maker then chooses the option that minimises his or her regret value, i.e. the option that maximises the regret-based perceived utility function. Therefore, in this paper, the maximum outcome value of each option is chosen as the reference point for calculating the regret value.

4.1.3 The regret-based perceived utility functions

Utility in regret theory is the degree of regret a decision maker may feel after making a choice. This level of regret can be thought of as the negative emotion or sense of loss associated with the difference between the actual choice and the other possible choices. The higher the utility value, the greater the advantage of the chosen option over other possible options, and thus the less regret the decision maker may feel in subsequent comparisons.

In regret theory, the decision maker's goal is usually to minimise regret, i.e., to choose an option that minimises regret. This approach considers the attitudes of decision makers in the face of uncertainty content and minimises the regret that may be felt as a result of making a particular choice. Utility is therefore an important indicator used in

Table 5 The global preference value

	C_1	C_2	C_3	C_4
s_1	{0}	{0.0, 0.0, 0.0029, 0.0328, 0.0928}	{0, 0.0043, 0.017, 0.0307, 0.1115}	{0, 0.0047, 0.0166, 0.1135}
s_2	{0.0076, 0.0109, 0.0316, 0.0671, 0.1303}	{0}	{0.0109, 0.0377, 0.0377, 0.0752, 0.1977}	{0, 0.0032, 0.0247, 0.1306}
s_3	{0, 0.0024, 0.0172, 0.0377}	{0, 0.0029, 0.0065, 0.015, 0.026}	{0}	{0, 0.0062, 0.017, 0.0254}
s_4	{0.0299, 0.0661, 0.096, 0.125, 0.1605}	{0, 0.0029, 0.0752, 0.1108, 0.1176}	{0.0661, 0.0938, 0.1145, 0.1507, 0.1507}	{0}

Table 6 The overall loss/gain value

	π^-	π^+
s_1	$\{0.0373, 0.0763, 0.1267, 0.1978, 0.2974\}$	$\{0, 0.0043, 0.0245, 0.0781, 0.2854\}$
s_2	$\{0, 0.0058, 0.0839, 0.1529, 0.2203\}$	$\{0.0184, 0.0482, 0.0711, 0.1586, 0.3934\}$
s_3	$\{0.0763, 0.1317, 0.1624, 0.2387, 0.3946\}$	$\{0, 0.0029, 0.015, 0.0484, 0.0865\}$
s_4	$\{0, 0.014, 0.0572, 0.2489\}$	$\{0.094, 0.1562, 0.2597, 0.3392, 0.3709\}$

Table 7 The relative outcome function of s_1

	S_k	$\neg S_k$
A_p	$\{0, 0.0043, 0.0245, 0.0781, 0.2854\}$	$\{0\}$
A_b	$\{0, 0.0022, 0.0123, 0.0391, 0.1427\}$	$\{0.0186, 0.0382, 0.0634, 0.0989, 0.1487\}$
A_n	$\{0\}$	$\{0.0373, 0.0763, 0.1267, 0.1978, 0.2974\}$

regret theory to assess the pros and cons of different options in order to guide the decision maker towards the best choice. Next, I will introduce the method of finding the perceived utility function on the basis of regret theory.

The relative outcome function $O^i_{\triangle\heartsuit}(\triangle = \{p, b, n\}; \heartsuit = \{p, n\})$ of object i can be determined from the relative outcome matrix described in Section 3.1.1 above, where $\{O^i_{pp}, O^i_{bp}, O^i_{np}\}$ and $\{O^i_{pn}, O^i_{bn}, O^i_{nn}\}$ represent the relative outcome function when the object e_i belongs to the decision class S and when it belongs to the decision class $\neg S$, respectively, when taking action $\{A_p, A_b, A_n\}$.

First, we calculate the direct utility function without considering the regret value, as shown in Equation 11.

$$U^i_{\triangle\heartsuit} = \frac{1 - e^{-\theta \cdot O^i_{\triangle\heartsuit}}}{\theta}, \tag{11}$$

where $\theta \in [0, +\infty)$ is the risk aversion parameter.

The model proposed by [27] calculates the regret function as always non-positive, which hinders subsequent calculations. Therefore, we propose a new regret-based perceived utility function.

In regret theory, the decision maker aims to minimize the regret value of choosing a particular object without choosing the reference object. This is equivalent to making the regret value of choosing the reference object without choosing the particular object mentioned the highest. In this case, the higher the regret value, the better the particular object is.

The maximum outcome value for each object is selected as the reference point for calculating the regret value, as discussed in Section 3.1.2.

$$O^i_{\star} = max\{O^i_{\triangle\heartsuit}\}. \tag{12}$$

The utility difference of the reference object $O^i_{\star}$ with respect to the particular object $O^i_{\triangle\heartsuit}$ is:

$$\bigtriangleup U^i_{\triangle\heartsuit} = U^i_{\star} - U^i_{\triangle\heartsuit}. \tag{13}$$

The regret value caused by the utility difference is as in Equation 14:

$$R(\bigtriangleup U^i_{\triangle\heartsuit}) = 1 - e^{-\delta \cdot \triangle U^i_{\triangle\heartsuit}}, \tag{14}$$

where $\delta \in [0, +\infty)$ is the regret aversion parameter.

Definition 1.1 Assuming the outcomes of alternatives o^i_1, o^i_2, ... ,o^i_m are O^i_1, O^i_2, ... ,O^i_m respectively.The regret-based perceived utility function is as follows:

$$MU^i_{\triangle\heartsuit} = U^i_{\triangle\heartsuit} + R(U^i_{\star} - U^i_{\triangle\heartsuit}), \tag{15}$$

where $R(U^i_{\star} - U^i_{\triangle\heartsuit})$ is always non-negative, and a higher regret function corresponds to a higher the regret-based perceived utility function.

Example 1.2 (Continued with Example 3.1)

We suppose that the risk aversion parameter $\theta = 0.6$ and the regret aversion parameter $\delta = 0.4$.

The sixth step is to calculate the direct utility function for s_1 from the relative outcome O^1_{pp}.

$$\begin{aligned} U^1_{pp} = \{&\tfrac{1-e^{-0.6*0}}{0.6}, \tfrac{1-e^{-0.6*0.0043}}{0.6}, \tfrac{1-e^{-0.6*0.0245}}{0.6}, \\ &\tfrac{1-e^{-0.6*0.0781}}{0.6}, \tfrac{1-e^{-0.6*0.2854}}{0.6}\} \\ = \{&0, 0.0043, 0.0243, 0.0763, 0.2623\}. \end{aligned}$$

Similarly, we can compute the direct utility function of s_1 in other actions and states.

$$\begin{aligned} U^1_{pn} &= \{0\}; \\ U^1_{bp} &= \{0, 0.0022, 0.0123, 0.0386, 0.1368\}; \\ U^1_{bn} &= \{0.0185, 0.0378, 0.0622, 0.096, 0.1423\}; \\ U^1_{np} &= \{0\}; \\ U^1_{nn} &= \{0.0369, 0.0746, 0.122, 0.1865, 0.2724\}. \end{aligned}$$

Immediately after this, the utility difference of s_1 is calculated as shown in Table 8.

The seventh step is to calculate the regret function.

$$\begin{aligned} R(\bigtriangleup U^1_{pp}) = \{&1 - e^{-0.4*0.0369}, 1 - e^{-0.4*0.0703}, 1 - e^{-0.4*0.0977}, \\ &1 - e^{-0.4*0.1102}, 1 - e^{-0.4*0.0101}\} \\ = \{&0.0147, 0.0277, 0.0383, 0.0431, 0.004\}. \end{aligned}$$

The eighth step is to calculate the regret-based perceived utility function.

$$\begin{aligned} MU^1_{pp} = \{&0 + 0.0147, 0.0043 + 0.0277, 0.0243 \\ &+0.0383, 0.0763 + 0.0431, 0.2623 + 0.004\} \\ = \{&0.0147, 0.032, 0.0626, 0.1194, 0.2663\}. \end{aligned}$$

Table 8 The utility difference of s_1

	S	$\neg S$
A_p	$\{0.0369, 0.0703, 0.0977, 0.1102, 0.0101\}$	$\{0.0369, 0.0746, 0.122, 0.1865, 0.2724\}$
A_b	$\{0.0369, 0.0724, 0.1097, 0.1479, 0.1356\}$	$\{0.0184, 0.0368, 0.0598, 0.0905, 0.1301\}$
A_n	$\{0.0369, 0.0746, 0.122, 0.1865, 0.2724\}$	$\{0\}$

Table 9 The regret-based perceived utility function of s_1

	S	$\neg S$
A_p	$\{0.0147, 0.032, 0.0626, 0.1194, 0.2663\}$	$\{0.0147, 0.0294, 0.0476, 0.0719, 0.1032\}$
A_b	$\{0.0147, 0.0307, 0.0552, 0.096, 0.1896\}$	$\{0.0258, 0.0524, 0.0858, 0.1316, 0.193\}$
A_n	$\{0.0147, 0.0294, 0.0476, 0.0719, 0.1032\}$	$\{0.0369, 0.0746, 0.122, 0.1865, 0.2724\}$

Similarly, the regret-based perceived utility function for s_1 in other actions and states could be calculated as given in Table 9.

4.2 Satisfaction-based weighting function

While some progress has been made in the field of hesitant fuzzy set research, there is still a paucity of methodologies for stochastic multi-attribute decision-making in hesitant fuzzy environments.

Based on prospect theory, [57] ranks the objects by calculating the probability weights and prospect value functions in different states. Although her results take psychological factors into account, there are problems such as more parameters involved in the calculation and the need to provide reference points beforehand.

In order to address the aforementioned limitations, this paper employs the proposed hesitant fuzzy stochastic multi-attribute decision-making methodology, developed by [20], which is founded upon the principles of regret theory.In order to consider the psychological factors of decision makers, it is first necessary to calculate group satisfaction. This is used to construct the weight optimisation model. The expected utility is then calculated to rank the objects.

Let $v = \bigcup_{l=1}^{|v|}\{v^{\psi(l)}\}$ be a hesitant fuzzy element,then the satisfaction index of the decision group is:

$$\Phi(v) = \frac{S(v)}{1+V(v)} = \frac{\frac{1}{|v|}\sum_{l=1}^{|v|} v^{\psi(l)}}{1+\frac{1}{|v|}\sum_{l=1}^{|v|}\left|v^{\psi(l)} - S(v)\right|}, \tag{16}$$

where $v^{\psi(l)}$ denotes the lth smallest element of v, $|v|$ denotes the number of elements in v, $S(v)$ denotes the score function for v, reflecting the overall characteristics of v. The larger $S(v)$ is, the higher the satisfaction; $V(v)$ denotes the mean deviation function for v , indicating the extent of discord within the decision-making group; the smaller $V(v)$ is, the more unanimous is the group's opinion and the higher the satisfaction.

Assuming that $\triangle = \{p, b, n\}$ represents the set of choices, $\heartsuit = \{p, n\}$) represents the set of object events, and the perceived utility function of each choice for each event is MU, the weighting function for the respective events is modelled as follows:

$$(\mathrm{M}-1)\begin{cases} \max F(W_\vartheta) = \sum_{\varrho\in\Delta}\sum_{\vartheta\in\heartsuit}\Phi(MU_{\varrho\vartheta})W_\vartheta, \\ \text{s.t. } \sum_{\vartheta\in\heartsuit} W_\vartheta = 1, 0 \leqslant W_\vartheta \leqslant 1. \end{cases}$$

Solve for the event weight vector as:

$$W_\vartheta = \frac{\sum_{\varrho\in\Delta}\Phi(MU_{\varrho\vartheta})}{\sqrt{\sum_{\vartheta\in\heartsuit}(\sum_{\varrho\in\Delta}\Phi(MU_{\varrho\vartheta}))^2}}.$$

Since the weights must satisfy the conditions$\sum_{\vartheta\in\heartsuit} W_\vartheta = 1$, they are normalised. Then satisfaction-based weighting function is shown in Equation 17.

$$W_\vartheta^{\cdot} = \frac{W_\vartheta}{\sum_{\vartheta\in\heartsuit} W_\vartheta}. \tag{17}$$

Example 1.3 (Continued with Example 3.2)

The ninth step is to calculate the satisfaction for each event from the perceived utility function.

The score function and the mean deviation function are calculated first.

$$S(MU_{pp}^1) = \frac{1}{5} * (0.0147 + 0.032 + 0.0626 + 0.1194 + 0.2663) = 0.099,$$

$$V(MU_{pp}^1) = \frac{1}{5} * (|0.0147 - 0.099| + |0.032, -0.099| + |0.0626 - 0.099| + |0.1194 - 0.099| + |0.2663 - 0.099|) = 0.0749.$$

Then calculate the satisfaction level according to Equation 16.

$$\Phi(MU_{pp}^1) = \frac{S(MU_{pp}^1)}{1+V(MU_{pp}^1)} = \frac{0.099}{1+0.0749} = 0.0921.$$

Similarly the satisfaction of s_1 in different states and actions can be calculated as shown in the Table 10.

Table 10 The satisfaction of s_1

	S	$\neg S$
A_p	$\{0.0921\}$	$\{0.0519\}$
A_b	$\{0.0734\}$	$\{0.0929\}$
A_n	$\{0.0519\}$	$\{0.1291\}$

The tenth step is to calculate the weight of each event and normalise it.

$$\sum_{\varrho\in\Delta}\Phi(MU^1_{\varrho p}) = 0.0921 + 0.0734 + 0.0519 = 0.2174,$$

$$\sum_{\varrho\in\Delta}\Phi(MU^1_{\varrho n}) = 0.0519 + 0.0929 + 0.1291 = 0.2739;$$

$$W^1_p = \frac{0.2174}{\sqrt{(0.2174)^2 + (0.2739)^2}} = 0.6217,$$

$$W^1_n = \frac{0.2739}{\sqrt{(0.2174)^2 + (0.2739)^2}} = 0.7833.$$

The weights of s_1 for state *S* are obtained after normalisation as follows:

$$W^{\cdot 1}_p = \frac{0.6217}{0.6217 + 0.7833} = 0.4425.$$

Similarly, the weights of other objects under different events can be calculated as shown in the Table 11.

4.3 RT-based expected utility function in HF environment

Regret theory defines utility value as the decision maker's subjective evaluation of each option's value or satisfaction, used to measure the degree of preference or importance. A higher utility value indicates greater satisfaction or expected value of the option. To minimize possible regrets, decision makers must assess the utility of each option. The utility value in this context refers to the level of regret that may arise from a decision, specifically the negative emotions or feelings of loss associated with the differences between the chosen option and other possible options. Therefore, options with high utility values are more advantageous than other options, reducing the likelihood of decision maker regret.

Similarly, in accordance with the RT-based TWD model in the HF environment, each object is required to select the action that exhibits the highest utility value. Thus, we define a type of expected utility function based on regret theory in a HF environment. The objects are then sorted and classified according to this expected utility function.

Table 11 The weights under different events

	$W^{\cdot}_p$	$W^{\cdot}_n$
s_1	$\{0.4425\}$	$\{0.5575\}$
s_2	$\{0.5346\}$	$\{0.4654\}$
s_3	$\{0.389\}$	$\{0.611\}$
s_3	$\{0.5958\}$	$\{0.4042\}$

Definition 1.2 Assuming that the object has three actions ($\Delta = \{p, b, n\}$) and two states ($\heartsuit = \{p, n\}$),the expected utility function under each action is:

$$\begin{aligned} EMU_p &= W^{\cdot}_p MU_{pp} \oplus W^{\cdot}_n MU_{pn};\\ EMU_b &= W^{\cdot}_p MU_{bp} \oplus W^{\cdot}_n MU_{bn};\\ EMU_n &= W^{\cdot}_p MU_{np} \oplus W^{\cdot}_n MU_{nn}, \end{aligned} \tag{18}$$

where $MU_{\Delta\heartsuit}$ is the perceived utility function of each behaviour for each event (state), and $W^{\cdot}_{\heartsuit}$ is the weight of each event.

The decision rules for maximising expected utility are show as:

$(DR_p) EMU^i_p \geq EMU^i_b$ and $EMU^i_p \geq EMU^i_n \Rightarrow s_i \in Pos(S)$;
$(DR_b) EMU^i_b \geq EMU^i_p$ and $EMU^i_b \geq EMU^i_n \Rightarrow s_i \in Bnd(S)$;
$(DR_n) EMU^i_n \geq EMU^i_p$ and $EMU^i_n \geq EMU^i_b \Rightarrow s_i \in Neg(S)$.

The argument domain *U* is partitioned into three separate non-overlapping regions: the positive region (*Pos*(*S*)), the negative region (*Neg*(*S*)) and the boundary region (*Bnd*(*S*)), where $U = Pos(S) \cup Bnd(S) \cup Neg(S)$ and $\emptyset = Pos(S) \cap Bnd(S) \cap Neg(S)$.

The ranking rules are show as:

$(SR_1) Pos(S) > Bnd(S) > Neg(S)$;
$(SR_2) EMU^i > EMU^j \Rightarrow s_i > s_j$,

where means that objects in the positive region are sorted before those in the boundary region, and objects in the boundary region are sorted before those in the negative region; if the two objects are in a region, the higher the expected utility function, the higher the ordering.

Example 1.4 (Continued with Example 3.3)

The eleventh step is to calculate the expected utility function.

$$\begin{aligned} EMU^1_p =& W^{\cdot}_p MU_{pp} \oplus W^{\cdot}_n MU_{pn}\\ =&\{0.0065, 0.0142, 0.0277, 0.0528, 0.1178\}\\ &\oplus\{0.0082, 0.0164, 0.0265, 0.0401, 0.0575\}\\ =&\{0.0146, 0.0303, 0.0535, 0.0908, 0.1686\}. \end{aligned}$$

The expected utility function of s_1 under the other two actions is as follows:

$$EMU^1_b = \{0.0208, 0.0424, 0.0711, 0.1127, 0.1825\},$$
$$EMU^1_n = \{0.0269, 0.0541, 0.0876, 0.1325, 0.1906\}.$$

Since $EMU^1_n \geq EMU^1_p$ and $EMU^1_n \geq EMU^1_b$, s_1 belongs to the class Neg(S). Similarly, classifying several other objects results in: $s_2, s_4 \in Pos(S)$, $s_1, s_3 \in Neg(S)$, where $EMU_p^4 > EMU_p^2$ and $EMU_n^1 > EMU_n^3$, so $s_4 > s_2$ and $s_1 > s_3$. The final ranking result is $s_4 > s_2 > s_3 > s_1$.

4.4 Algorithms for RT-based TWD modelling in HF environment

In this section, we present the algorithms of the RT-based TWD model Algorithm 1 in the HF environment and analyse the complexity of the model based on the algorithm.

Input: An HF environment H, $w = \{w_1, w_2, \ldots, w_k\}^T$, the degree of dominance ρ, the utility coefficient ξ, the risk aversion parameter θ, and the regret aversion parameter δ.

Output: Classification results, ranking results.

1 **for** $s_i \in \mathbb{S}$ *and* $C_k \in \mathbb{C}$ **do**
2   **calculate:** the HF deviation $D_k(s_i, s_j)$ and the hesitant preference degree $P_k(s_i, s_j)$. // by Eq. (5) and Eq. (7)
3 **end for**
4 **for** $s_i \in \mathbb{S}$ *and* $C_k \in \mathbb{C}$ **do**
5   **calculate:** the global preference value $\pi(s_i, s_k)$. // by Eq. (8)
6 **end for**
7 **for** $s_i \in \mathbb{S}$ **do**
8   **calculate:** the overall loss value $\pi^-(s_i)$ and the overall gain value $\pi^+(s_i)$. // by Eq. (10) and Eq. (9)
9 **end for**
10 **for** $s_i \in \mathbb{S}$ **do**
11   **construct:** the relative outcome function.
12 **end for**
13 **for** $s_i \in \mathbb{S}$ **do**
14   **calculate:** the direct utility function $U^i_{\triangle\heartsuit}$ and the regret value $R(\triangle U^i_{\triangle\heartsuit})$. // by Eq. (11) and Eq. (14)
15 **end for**
16 **for** $s_i \in \mathbb{S}$ **do**
17   **calculate:** the regret-based perceived utility function $MU^i_{\triangle\heartsuit}$. // by Eq. (15)
18 **end for**
19 **for** $s_i \in \mathbb{S}$ **do**
20   **calculate:** the satisfaction $\Phi(v)$. // by Eq. (16)
21 **end for**
22 **for** $s_i \in \mathbb{S}$ **do**
23   **calculate:** the weight function W. // by Eq.(17)
24 **end for**
25 **for** $s_i \in \mathbb{S}$ **do**
26   **calculate:** the expected utility function EMU. // by Eq.(18)
27 **end for**
28 **for** $s_i \in \mathbb{S}$ **do**
29   **decision:** with the support of classification rules (DR_p, DR_b and DR_n).
30 **end for**
31 **for** $s_i \in \mathbb{S}$ **do**
32   **decision:** with the support of ranking rules (SR_1 and SR_2).
33 **end for**
34 **return:** Classification results and sorting results.

Algorithm 1 The RT-based TWD model under an HF environment

Remark 1.1 Step 1, Step 2, and Step 3 all have a time complexity of $O(m^2 n)$. The time complexity of step 4 is $O(m^2)$. The time complexity of step 5 is $O(m)$. The time complexity of step 6 is $O(mn)$. The time complexity of step 7 is $O(m)$. The time complexity of step 8 is $O(mn)$. Step 9 has a time complexity of $O(m)$. The time complexity of step 10 is $O(mn)$. Step 11 has a time complexity of $O(m)$. The time complexity of step 12 is $O(3m)$. In summary, the overall time complexity of Algorithm 1 is $O(m^2)$.

5 Case study

In this section, the model proposed in this paper will be used to address the problem of forest fires and the appropriateness and effectiveness of the method will be verified by comparing it with other methods.

5.1 The model's practical application to the issue of Algerian forest fires

The dangers of forest fires are extensive and far-reaching. First, they disrupt the ecological balance, burning vegetation on a large scale, causing soil erosion and water pollution, damaging biodiversity and threatening the survival of wildlife. Secondly, wildfires have a huge impact on human socio-economics, destroying crops, homes and infrastructure, causing property loss and unemployment. In addition, the smoke and noxious gases released by fires endanger human health, exacerbating respiratory and cardiovascular diseases. Finally, climate change and land degradation caused by forest fires affect the global environment and have a long-term impact on the Earth's ecosystem.

Forest fires are characterised by rapid spread and devastation. Due to the dense vegetation in forests, dry climatic conditions and windy environments, fires in forested areas tend to spread rapidly and develop into large fires. The fires are so intense that they cause widespread destruction of vegetation and soil, triggering disasters such as landslides and mudslides. In addition, forest fires are often affected by complex terrain and human activities, making them difficult to fight and resulting in high firefighting costs.

Preventing forest fires is therefore crucial. Not only is it necessary to strictly control fire sources, prohibit the use of fire in the wild, and limit behaviours such as burning waste. It is also necessary to establish a sound monitoring system and use satellite monitoring, drone technology and other means to detect fires in good time. More importantly, it is necessary to predict the likelihood of fires in advance in order to prevent fires, detect and control them as early as possible, and reduce the losses caused by fires.

The model presented in this paper is validated using the Algerian forest fire dataset. The Algerian Forest Fires Dataset, comprising objective data such as temperature, humidity, wind speed, and rainfall, is modelled using hesitant fuzzy sets (HFS) to capture uncertainty and ambiguity. Through the PROMETHEE-II method, the relative gains and losses for each object are calculated, thereby quantifying the decision-maker's preferences based on the dataset. The integration of regret theory facilitates a further linkage between the objective data and the psychological behaviour of the decision-maker, including regret aversion and risk perception. The model provides a new approach to the decision-making problem by considering natural factors to predict whether a fire will occur or not.

Algerian Forest Fires dataset, comprising 244 instances, regroups data from the Bejaia region, located in the north-east of Algeria, and the Sidi Bel Arab region, located in the north-west of Algeria. The time period is from June 2012 to September 2012. The dataset includes 11 conditional attributes and 1 decision attribute, and the 244 instances were classified into 'on fire' classes (138 instances) and 'not on fire' classes (106 instances).

We only consider the effect of natural factors on fires, so we take the day as an object and use only some of the conditional attributes: temperature(Temperature), relative humidity(RH), wind speed(Ws) and rainfall(Rain) to determine whether a fire will occur. Among these attributes, temperature and wind speed are positively correlated with the likelihood of fire occurrence, meaning that higher values of temperature and wind speed increase the probability of fire. These are defined as Positive Attributes. On the other hand, relative humidity and rainfall are negatively correlated with the likelihood of fire occurrence, meaning that higher values of humidity and rainfall decrease the probability of fire. These are defined as Negative Attributes. The evaluation values corresponding to Positive Attributes and Negative Attributes are shown in Table 12 and Table 13, respectively.

The use of hesitant fuzzy systems (HFNs) is motivated by their ability to effectively handle uncertainty and vagueness in real-world data, which is particularly relevant in the context of forest fire prediction, where environmental factors often have imprecise or unstable information. To convert the data types of Algerian Forest Fires into the HFNs schema, we preprocess the four selected conditional attributes. [23] article presents nine linguistic variables and their corresponding HFNs. Using this method, we can take the temperature attribute as an example. First, we normalise the values of all objects under the temperature attribute. Then, based on the normalised values, we can gain the evaluate values of each object under the temperature attribute. Similar preprocessing steps are applied to wind speed (Ws), relative humidity (RH), and rainfall (Rain) to obtain their respective evaluation values. Secondly, the entropy weight method [34] is utilised to derive weight vectors $w = [0.0039, 0.0182, 0.0098, 0.9681]$ for the four conditional attributes.

The individual parameter assumptions used in the analysis of Algerian Forest Fires dataset in this model are as follows: the degree of dominance of $\rho = 0.9$, the utility coefficient of $\xi = 0.417$, the risk aversion parameter $\theta = 0.5$ and the regret aversion parameter $\delta = 0.7$.

Remark 1.2 The result of ranking 244 objects in the dataset using this model is shown in Figure 2, and the classification results are shown in Figure 3. Figure 2 indicates that the optimal object is the 195th object, which implies that there is the highest probability of a fire occurring on 12 August 2012 in the Sidi-Bel Abbes Region. Conversely, the least optimal object is the 162nd object, indicating that there is the lowest probability of a fire occurring on 10 July 2012 in the Sidi-Bel Abbes Region. Figure 3 indicates that Pos(S) comprises 213 objects, signifying that there are 213 days when a fire occurs. Conversely, Bnd(S) comprises 2 objects, indicating that there are 2 days when a fire is likely to occur. Finally, Neg(S) comprises 29 objects, indicating that there are 29 days when a fire does not occur.

Table 12 Evaluation value of Positive Attributes

Normalized values		[0.00,0.15]	[0.15,0.30]	[0.30,0.40]	[0.40,0.55]	[0.55,0.70]	[0.70,0.85]	[0.85,1.00]
	Pessimist	0.00	0.15	0.30	0.40	0.55	0.70	0.85
evaluate values	Moderate	0.075	0.225	0.35	0.475	0.625	0.775	0.925
	Optimist	0.15	0.30	0.40	0.55	0.70	0.85	1.00

Table 13 Evaluation value of Negative Attributes

Normalized values		[0.00,0.15]	[0.15,0.30]	[0.30,0.40]	[0.40,0.55]	[0.55,0.70]	[0.70,0.85]	[0.85,1.00]
	Pessimist	0.85	0.70	0.55	0.40	0.30	0.15	0.00
evaluate values	Moderate	0.925	0.775	0.625	0.475	0.35	0.225	0.075
	Optimist	1.00	0.85	0.70	0.55	0.40	0.30	0.15

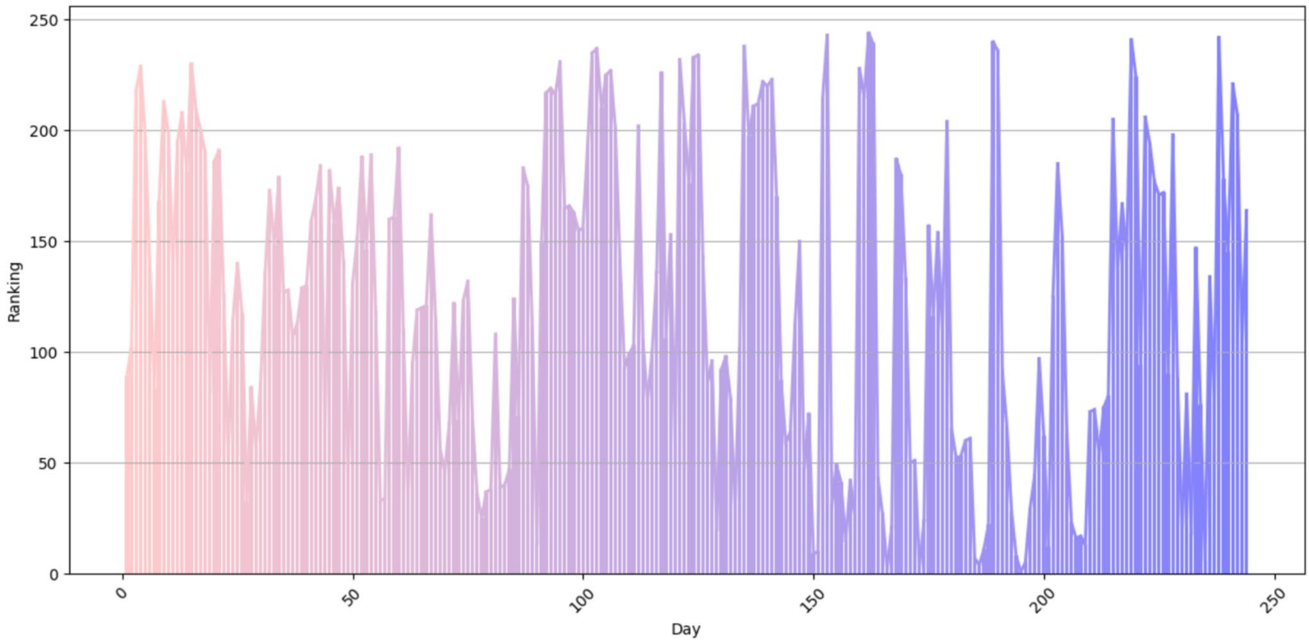

Fig. 2 The ranking results

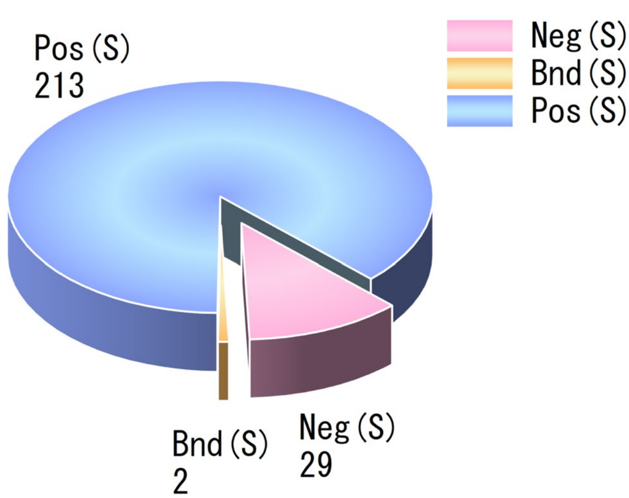

Fig. 3 The classification results

5.2 Comparative analysis

In this section, we will assess the rationality and validity of the method by comparing it to other methods.

5.2.1 Comparative evaluation of the ranking outcomes

[1] first proposed the theory of regret, a theory that has since been widely developed and applied in a variety of settings and scenarios. Regret theory explores the feelings of regret that can arise when faced with different choices and how such feelings of regret can be taken into account in the decision-making process. In addition to classical regret theory, a number of extended approaches and variants have emerged to better explain and manage the phenomenon of regret in different contexts.

In this section, we compare and analyse the ranking results of the proposed model with some classical approaches as well as some extended approaches. By analysing these ranking results, we can further validate the rationality of the proposed model.

The GHFWA [40] algorithm provides a way to combine information from different sources by aggregating data from multiple sources. The TOPSIS [42] method, on the other hand, focuses on the distance between the desired solution and the target solution in order to find the optimal solution in the decision-making process. The TODIM [54] method examines the psychological behaviour of the decision maker in order to better understand his/her preferences and behaviour patterns in the decision-making process. The TOPSIS method proposed by [39] uses probability theory (PT) integrated with TOPSIS to deal with uncertainty and risk. [28] integrated PT into the PROMETHEE method, specifically considering the impact of different risk attitudes on profit and loss. The ARAS method developed by [24] focuses on decision analysis and optimisation in RF environments and provides an effective solution for dealing with uncertainty and risk.

The method of [35] takes into account the psychological factors of the decision maker, gain and loss assessment, etc. and solves the problem of uncertainty in multi-attribute decision making.

The top five objects for each method are:

$$\begin{aligned} \text{Proposed model:} &\quad s_{195} > s_{166} > s_{173} > s_{186} > s_{196}; \\ \text{GHFWA:} &\quad s_{200} > s_{195} > s_{185} > s_{194} > s_{242}; \\ \text{TOPSIS:} &\quad s_{195} > s_{200} > s_{186} > s_{185} > s_{194}; \\ \text{TODIM:} &\quad s_{195} > s_{186} > s_{173} > s_{166} > s_{056}; \\ \text{Wang et al.'s model:} &\quad s_{195} > s_{200} > s_{230} > s_{186} > s_{185}; \\ \text{Tian et al.'s model:} &\quad s_{230} > s_{233} > s_{200} > s_{151} > s_{230}; \\ \text{Mishra's model:} &\quad s_{200} > s_{185} > s_{194} > s_{195} > s_{199}; \\ \text{Wang's model:} &\quad s_{195} > s_{166} > s_{173} > s_{186} > s_{196}. \end{aligned}$$

As can be seen from the ranking results, the optimal solution s_{195} in the model proposed in this paper also exhibits excellent performance in most of the existing methods. Specifically, the solution is ranked within the top 5% of all evaluated methods, demonstrating a consistently high level of performance. The optimal solution selected by this model has a stable ranking under multiple evaluation criteria without any significant downward trend, a finding that highlights the consistency and stability of the proposed model.

Since the plausibility of the eight models used for comparison has been demonstrated, this consistency and stability justifies the adequacy and applicability of the innovative method.

5.2.2 Comparative interpretation of the classification results

This section will present a comparative analysis of the classification results produced by the proposed model with those of other models. By examining these results, it will be possible to demonstrate the accuracy and reasonableness of the proposed model for classification.

[16] used a HF loss function to improve the reliability and accuracy of the method in complex situations; [17] evaluated the accuracy and robustness of the model through error analysis; [10] proposed a relative loss function based on an information table to provide a comprehensive and objective basis for decision making; [36] used a TWD model supported by PT to deal with uncertainty and risk uncertainty and risk; [15] Bayesian investment decision procedure integrates multiple factors; [34] super-ranked class method effectively classifies complex data; [11] proposed a rough set model based on the decision theory of fuzzy binary relations to resolve fuzzy uncertainty; [35] prospect theory-based multi-attribute TWD model takes into account subjective risk perception and provides a reliable basis for decision making.

Remark 1.3 As shown in Figure 4, the classification results of each method for 244 objects are as follows: The proposed model, 213 days to fire, 2 days to possible fire, and 29 days to no fire; Liang and Liu's model, 122 days to fire, 61 days to possible fire, and 61 days to no fire; Liang and Xu's model, 242 days to fire and 2 days to no fire; Jia and Liu's model, 168 days to fire, 15 days to possible fire, and 61 days to no fire; Wang and Li's model, 244 days to fire; Li and Huang's model, 244 days to fire; Wang and Ma's model, 234 days to fire, 6 days to possible fire, and 4 days to no fire; Jiang and Hu's model, 222 days to fire, and 22 days to no fire; Liang and Xu's model, 242 days to fire and 2 days to no fire; Wang's model, 217 days to fire, 15 days to possible fire, and 12 days to no fire.

In this section, we use CER (Classification Error Rate) to compare the performance of different classification models.

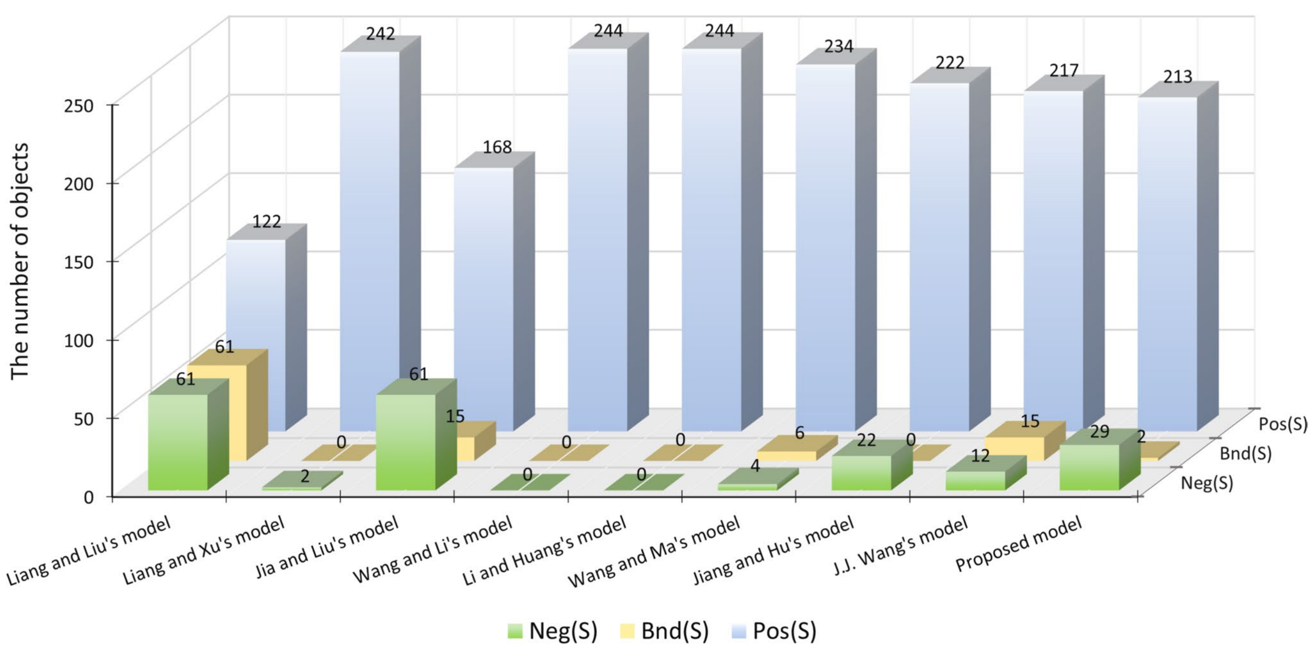

Fig. 4 The Classification of different methods

CER is one of the key metrics that can be used to assess the accuracy of a classification model. It measures the errors made by the model in performing the classification task, i.e., the proportion of samples that are misclassified. In practice, the CER can be the number of misclassified samples as a proportion of the total number of samples, or alternatively, as the number of misclassified samples as a proportion of the total number of samples in a particular category. For a classification model, accuracy is the proportion of samples that are correctly classified, while CER is the proportion of samples that are misclassified. Therefore, the lower the CER, the better the model performance and the higher the accuracy. The formula used in this paper to calculate the CER is as follows:

$$CER = \frac{N_{\neg S \to Pos(S)} + N_{S \to Neg(S)}}{244},$$

where $N_{\neg S \to Pos(S)}$ denotes that no fire occurs but is misclassified as POS(S) occurrence class; $N_{S \to Neg(S)}$ denotes that fire occurs but is misclassified as Neg(S) no fire class.

Remark 1.4 The CER of the different methods is shown in Figure 5. From the figure it is easy to obtain that, in Algerian Forest Fires dataset: the proposed model has a CER of 31.97%; Liang and Liu's model has a CER of 41.39%; Liang and Xu's model has a CER of 42.62%; Jia and Liu's model has a CER of 41.39%; Wang and Li's model has a CER of 43.44%; Li and Huang's model has a CER of 43.44%; Wang and Ma's model has a CER of 39.34%; Jiang and Hu's model has a CER of 34.43%; Wang's model has a CER of 33.61%; It can be observed that the CER obtained by our proposed model is the lowest in Algerian Forest Fires dataset, i.e., the investigated method demonstrated the most optimal performance and the highest classification accuracy. This also proves the reasonableness and accuracy of the innovative method.

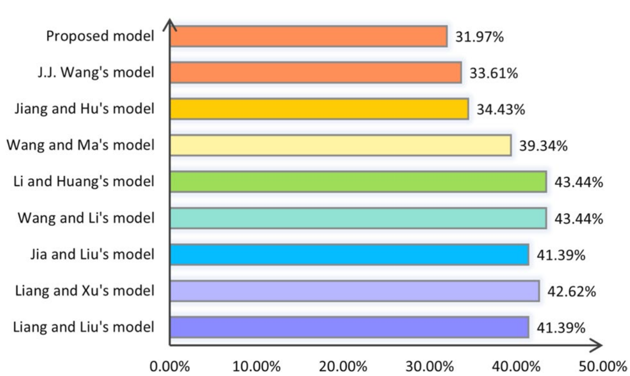

Fig. 5 The CER of different methods

5.2.3 Comparative evaluation under different data sets

Due to the disparate environments and conceptual frameworks within which each method is situated, their ranking and categorisation of the same dataset may diverge to some extent. To better illustrate the validity of the proposed model, we compare and analyse it in detail with a model that is also situated in a hesitant fuzzy environment - a three-way decision model based on prospect theory in a hesitant fuzzy environment [35].

As you can see in the Fig. 6, the order of the two models is very similar.

In order to facilitate a more intuitive comparison of the model effects, we employed the GHFWA [40] operator to quantify the degree of similarity between the two model ranking results.

$$\mathcal{S} = \frac{\sum_{i=1}^{n} a_i}{n}. \tag{19}$$

In addition, so as to confirm the rationality of the proposed model in depth, we conducted an exhaustive analysis of the correlation between the proposed model and Wang's model using the Spearman correlation coefficient [5].

$$\text{SRCC} = 1 - \frac{6\sum_{i=1}^{n}(a_i - b_i)_i^2}{n^3 - n}. \tag{20}$$

The correlation coefficient between the two model ranking results we calculated using the GHFWA operator method [40] is 0.9624; the value of Spearman's rank correlation coefficient [5] is 0.9227. This indicates the reasonableness of the proposed model.

When faced with decisions that cannot be clearly categorised or uncertain situations, delayed decision making can be an effective coping strategy that helps to reduce the error rate and improve the quality of decisions. In Section 4.2.2, the classification results of the model proposed by Jiajia Wang show that 15 days were classified as Likely to have a fire, which means that delayed decision-making was performed on 15 objects, none of which were able to obtain a clear classification judgement. In contrast, the classification results of our proposed model indicate that only two days were classified as possible fires, which means that only two objects failed to obtain a clear classification judgement.

Remark 1.5 As shown in Figure 7.When calculating the CER, 15 objects in Jiajia Wang's model made delayed decisions, which means that the classification error rate still reaches 33.61% when only 229 objects are subjected to a decision judgement. In contrast, the classification error rate of our proposed model is 31.97% when considering 242 objects. This indicates that our proposed model has

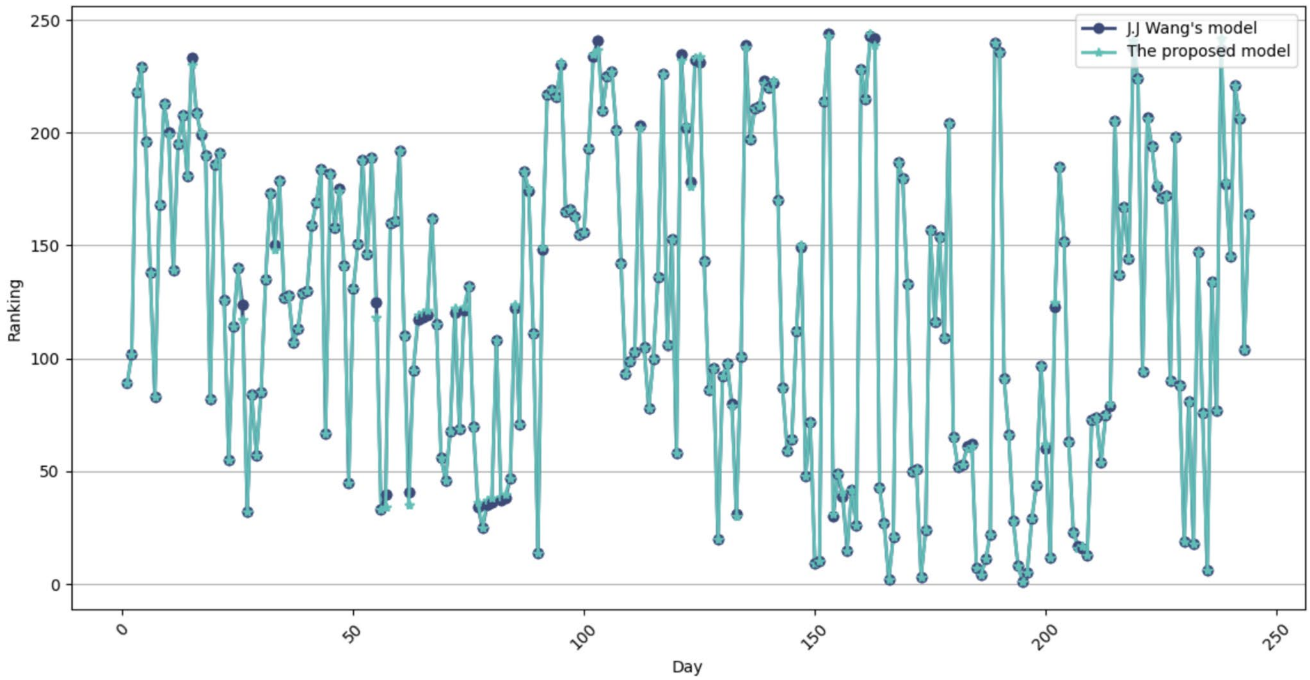

Fig. 6 Comparison with Prospect Theory-based Models in Ranking

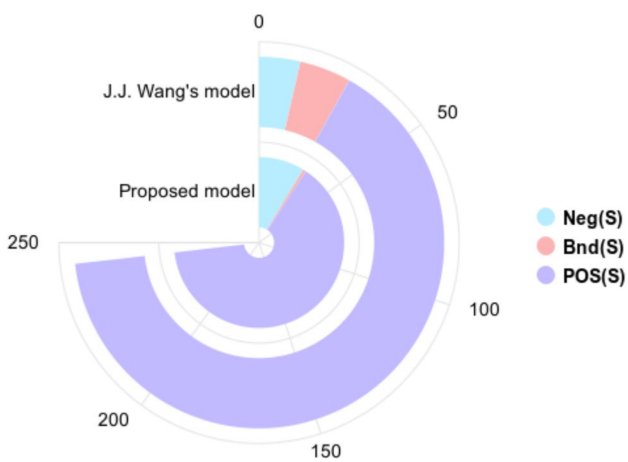

Fig. 7 Classification of Wang's model and proposed model

Table 14 Classification Error Rates (CER) of Different Methods on Various Datasets

	Cryotherapy	Cervical Cancer	Breast Cancer	Lung Cancer
Proposed model	**0.0222**	**0.0972**	**0.0386**	**0.0741**
J.J. Wang's method [35]	0.1444	0.1111	0.0472	0.0741
Jiang and Hu's method [11]	0.4778	0.1389	0	0.3333
Wang and Ma's method [34]	0.4222	0.1389	0.3104	0.5926
Li and Huang's method [15]	0.4667	0.5278	0	0.3333
Wang and Li's method [36]	0.4667	0.4444	0	0.3333
Jia and Liu's method [10]	0.3	0.2778	0.578	0.0741
Liang and Xu's method [17]	0.2667	0.1667	0.186	0.3333
Liang and Liu's method [16]	0.3111	0.5556	0.412	0.3333

improved in terms of accuracy and validity compared to Jiajia Wang's model. This result further consolidates the accuracy and superiority of the proposed model and provides stronger support for its feasibility in practical applications.

5.2.4 Comparative analysis under different data sets

To further validate the effectiveness and generalizability of our proposed model, we conducted extensive experiments on four additional datasets: Cryotherapy, Cervical Cancer Behavior Risk, Breast Cancer Wisconsin (Original), and Lung Cancer. These datasets cover a wide range of real-world scenarios, allowing us to evaluate the performance of our model in diverse contexts. The classification error rates (CER) of our proposed model and eight existing methods are summarized in Table 14. Figure 8 visually compares the performance of our proposed model and existing methods across datasets, highlighting its lower error rates and better generalization.These results highlight the superior performance of our proposed model in terms of classification accuracy and generalizability.

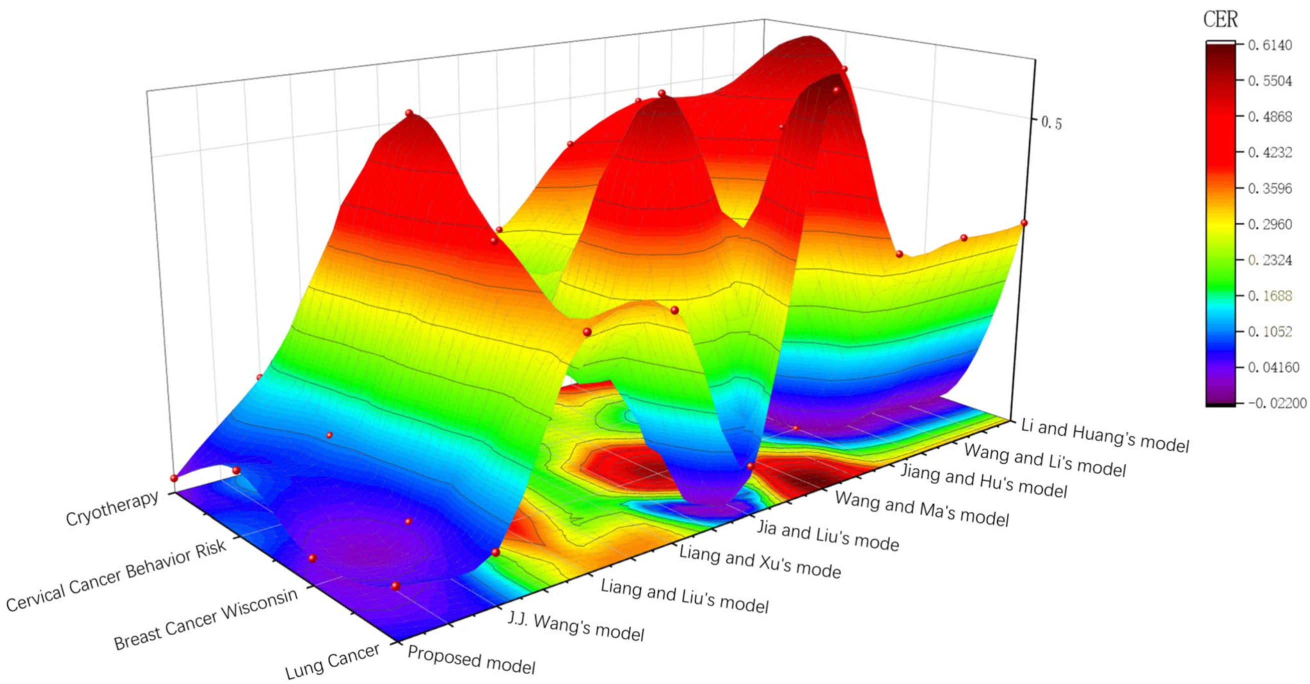

Fig. 8 CER of Different Methods on Various Datasets

Remark 1.6 The results demonstrate that our proposed model consistently achieves the lowest CER across all datasets. Specifically: On the Cryotherapy dataset, our model achieves a CER of 0.0222, significantly lower than the second-best method (J.J. Wang's method, CER = 0.1444). On the Cervical Cancer dataset, our model achieves a CER of 0.0972, outperforming all existing methods. On the Breast Cancer dataset, our model achieves a CER of 0.0386, demonstrating its robustness in handling complex data. On the Lung Cancer dataset, our model achieves a CER of 0.0741, comparable to the best existing method (J.J. Wang's method, CER = 0.0741).

Our proposed model achieves the lowest CER on all datasets, demonstrating its superior performance. To further validate the significance of our results, we conducted statistical analyses. For datasets where the CER data follows a normal distribution (Cryotherapy and Breast Cancer), we used parametric tests (F-tests). For datasets where the CER data does not follow a normal distribution (Cervical Cancer and Lung Cancer), we used non-parametric tests (Kruskal-Wallis tests):

$$\begin{array}{rl} \text{Cryotherapy}: & \text{ANOVA F} - \text{test results in F} = 5.23, \\ & \text{p} = 0.003; \\ \text{Cervical Cancer Behavior Risk}: & \text{Kruskal} - \text{Wallis test results in H} = 12.45, \\ & \text{p} = 0.014; \\ \text{Breast Cancer Wisconsin}: & \text{ANOVA F} - \text{test results in F} = 4.78, \\ & \text{p} = 0.008; \\ \text{Lung Cancer}: & \text{Kruskal} - \text{Wallis test results in H} = 10.89, \\ & \text{p} = 0.028. \end{array}$$

The p-values were all below 0.05, indicating that the improvements are statistically significant. The statistical analyses confirm that the robustness and generalizability of our approach in handling diverse real-world scenarios.

5.3 Discussion

5.3.1 Distinctions of the new model in contrast to current models

The commonality between the proposed model and the existing models is that they are all models for decision making in uncertain environments, where the reliability and accuracy of decisions are improved by various means. They all take into account different degrees of uncertainty and adopt strategies to deal with this uncertainty, resulting in more reliable and robust decision outcomes. As each model presents decisions differently, in this section we compare the differences between the models in terms of ranking, classification and the characteristics of the models themselves.

In order to facilitate a more intuitive understanding of the distinctions between the various methods, we have combined our method with 14 other methods and organised them in Table 15.

(1) Models in existence which have a ranking capabilities are:GHFWA [40],TOPSIS [42],TODIM [54],Wang et.al's model [39],Tian et.al's model [28],Mishra's model [24],Liang and Liu's model [16],Jia and Liu's model [10],Wang and Ma's model [34],Jiang and Hu's model [11] and Wang's model [35].

Table 15 Differences between the proposed model and existing models

	Psychological behaviour	Risk appetite	Loss/Utility/Value function	Ranking	Classification
Proposed model	✓	✓	✓	✓	✓
GHFWA [40]	×	×	×	✓	×
TOPSIS [42]	×	×	×	✓	×
TODIM [54]	✓	×	×	✓	×
Wang et.al's model [39]	✓	×	×	✓	×
Tian et.al's model [28]	✓	✓	×	✓	×
Mishra's model [24]	×	×	×	✓	×
Liang and Liu's model [16]	×	✓	✓	✓	✓
Liang and Xu's model [17]	×	✓	✓	×	✓
Jia and Liu's model [10]	×	✓	✓	✓	✓
Wang and Li's model [36]	✓	✓	✓	×	✓
Li and Huang's model [15]	×	✓	✓	×	✓
Wang and Ma's model [34]	×	✓	✓	✓	✓
Jiang and Hu's model [11]	×	✓	✓	✓	✓
Wang's model [35]	✓	✓	✓	✓	✓

Here we compare the Ranking principles of only some of the methods. GHFWA: Comprehensively considers multiple data sources and ranks them by aggregating their information; TOPSIS: Focuses on the distance between alternatives and the ideal solution and ranks them based on their relative proximity; TODIM: analyses the subjective preferences and cognitive characteristics of decision makers, calculates the degree of dominance and then ranks them in accordance with the value of the total prospect; Wang et al's model: consideration of the separation between the object and the positive and negative ideal solutions, and ranking based on the calculated gain/loss values; Tian et al's model: combines HF language with probability theory to examine the effect of varying risk attitudes on gain and loss, and then ranks them according to flow; Mishra's model: ranking based on overall performance scores; Wang's model: combines PT with TWD and ranks them according to expected prospect value; the proposed model: combines RT with TWD and ranks them according to expected utility value; the proposed model: combines RT with TWD and ranks them according to expected utility value.

(2) Models in existence which have a classification capabilities are: Liang and Liu's model [16],Liang and Xu's model [17],Jia and Liu's model [10],Wang and Li's model [36],Li and Huang's model [15],Wang and Ma's model [34], Jiang and Hu's model [11] and Wang's model [35].

Liang and Liu's model, Liang and Xu's model, Jia and Liu's model, Wang and Ma's model, and Jiang and Hu's model tend to select less risky objects for classification by introducing hesitant fuzzy loss functions, using error analysis, using relative loss functions, using probabilistic methods, and using Bayesian methods combined with TWD. Li and Huang's model integrates various factors such as benefits and costs, by applying Bayesian method to investment decision, the model is classified according to the expected profit. Wang and Li's model and Wang's model are both based on the TWD method of PT and classify the models according to the expected prospect value. The proposed model is based on the TWD method of RT and classifies the models according to the expected utility value.

(3) GHFWA [40], TOPSIS [42], Wang et.al's model [39], M-ishra's model [24],Liang and Liu's model [16],Jia and Liu's model [10],Jiang and Hu's model [11] Liang and Xu's model [17] and Li and Huang's model [15] focus more on the use of mathematical models or algorithms to analyze decision problems, taking into account different decision metrics, weight assignments, etc., but there may be less consideration of the psychological behavior of the decision maker. Although TODIM [54] and Wang et.al's model [39] take into account the psychological behaviour of the decision maker, their approaches to estimating target gains and losses differ from the proposed model. The former estimates gain and loss values based on a measurement function with dominance support, while the latter uses proximity to gauge the gain and loss of an object. Tian et.al's model [28] and Wang and Li's model [36],Nare based on prospect theory models, where Tian et.al's model is proposed in HFIS, while the background of Wang and Li's model is fuzzy

environment. Wang's model [35] was built based on RT in the HF environment, considering psychological factors while using objective data. The proposed model combines TWD with RT and considers both utility and psychological factors, and adds the consideration of potential regret in the decision process to make it more relevant to the real problem.

In summary, although all of these methods attempt to solve decision problems, they may differ in their focus, approach, and scope. Therefore, in decision-making practice, the appropriate method can be selected according to the specific situation and combined with the proposed model to analyze and solve the decision-making problem more comprehensively.

Remark 1.7 From Table 15: (1) TODIM [54], Wang et.al's model [39], Tian et.al's model [28], Wang and Li's model [36], Wang's model [35] and the proposed model take psychological behavior into account. Liang and Liu's model [16], Liang and Xu's model [17], Jia and Liu's model [10], Wang and Li's model [36], Li and Huang's model [15], Wang and Ma's model [34], Jiang and Hu's model [11] and Wang's model [35] and the proposed model both added Risk Appetite and used the Loss/Utility/Value function.

(2) GHFWA [40], TOPSIS [42], TODIM [54], Wang and Ma's model [34], Tian et.al's model [28] and Mishra's model [24] are limited to the ability to rank objects, whereas Liang and Xu's model [17], Wang and Li's model [36] and Li and Huang's model [15] are constrained to the capacity to classify objects. Liang and Liu's model [16], Jia and Liu's model [10], Wang and Ma's model [34], Jiang and Hu's model [11] and Wang's model [35] and the investigated model have both ranking and classification functions.

5.3.2 Strengths of the innovative model

1. Some approaches fail to consider the emotional responses of decision-makers when evaluating the outcomes or states of a given event in comparison to the selected alternative. In contrast, our approach integrates the RT model with the TWD model, which fully accounts for the influence of emotions on decision-making choices, thereby conferring decisions with greater realism and depth. This comprehensive consideration renders our method more valuable and feasible in practical applications.
2. The proposed model is a TWD model in hesitant fuzzy environment. The ability to consider various uncertainty and ambiguity factors in a hesitant-fuzzy environment makes the decision-making process more resilient and adaptable. This model is more realistic because many real-world decision-making problems are characterized by uncertainty and ambiguity. Furthermore, by considering the preferences and attitudes of different decision-makers, the accuracy and effectiveness of decision-making can be enhanced, while the risk and error of decision-making can be reduced.
3. Loss/Utility/Value function are usually determined subjectively in traditional methods, which may seem too arbitrary. The model proposed in this paper effectively provides an objective estimation of the outcome value of the object by using the PROMETHEE-based method, which significantly reduces the influence of subjective factors on decision making and improves the objectivity and scientificity of decision making. Compared with traditional methods, in which subjective determination is often easily affected by individual bias or experience limitations. The model in this paper, on the other hand, relies more on data and mathematical algorithms, especially the flow values based on the PROMETHEE method, which greatly reduces the possibility of bias and makes the decision more fair and objective. This model not only provides a more reliable basis for decision making, but also reduces the risk and uncertainty of decision making. By basing decisions more reliably on outcome values, decision makers are able to choose the best option with greater confidence, thus improving the overall quality of decision making.
4. In the traditional method, the perceived utility function is always non-positive. In contrast, the new perceived utility function established in this paper is always non-negative, which is more adaptable to the calculation of subsequent weights, the expected utility value, and other content.
5. While existing methods typically focus solely on either ranking or classification, real-world decision-making often requires both capabilities, especially when dealing with a large number of objects. Our method addresses this need by integrating classification and ranking into a unified framework. Specifically, it first classifies objects into distinct categories based on specific rules and then ranks objects within each category according to their characteristics. This two-step approach not only provides a structured decision-making process, but also increases the practicality and accuracy of the results. By combining classification and ranking capabilities, our method better meets the needs of decision makers and provides a more comprehensive and flexible tool for handling complex decision scenarios.
6. According to Figure 5, it shows that the proposed model has the lowest classification error rate compared to other models. This result clearly shows the high accuracy and effectiveness of the proposed model in the classification task. This low error rate implies that the model performs

well in correctly categorizing the data, which further proves the feasibility and effectiveness of the model in practical applications.

6 Experimental analysis and evaluations

In decision problems, different parameter values correspond to different results, and it is of paramount importance to idenitify and utilise appropriate parametrically values; improper parameter selection may lead to abnormal classification results. Therefore, this section analyses the effect of different parameter values on the decision results so that the decision maker can select more reasonable parameters to obtain the classification results and ranking results. Four parameters are considered in this paper: the dominance degree ρ, the utility pursuit coefficients ξ, the risk aversion parameter θ and the regret aversion parameter δ.

6.1 Experimental investigation of the prevalence level

As illustrated in Equation 7, the dominance degree ρ represents a pivotal variable in the computation of hesitant preference. It is therefore necessary to consider the impact of the value of the parameter p on the results. In practical problems, the dominance degree of each attribute will depend on the preferences of the decision-maker. For the sake of simplicity, it is assumed that each attribute has the same ρ -value.

We start by setting the values of the other parameters: $\xi = 0.417$; $\theta = 0.5$; $\delta = 0.7$. The value of ρ is in the range $[0.5, 1]$, and for the 244 objects in Algerian Forest Fires dataset, we set the step size to 0.1 and selected five different dominance degree. The corresponding ranking results for each dominance degree are shown in Figure 9.

Remark 1.8 As illustrated in Figure 9, when the parameters ξ, θ and δ are maintained at a constant value and the ρ parameter is incrementally increased, the distinct parameter values yield comparable sorting outcomes, with the optimal object consistently identified as s_{195}. This indicates that parameter variations have a minimal effect on the optimal results and the overall ranking results. This evidence supports the stability of the proposed ranking.

In addition to verifying the plausibility and stability of the models, we tend to choose more efficient models, so we compared the classification results under different degrees of dominance and chose the parameters that would make the models perform better based on the classification error rate. The classification results and CERs under different dominance degrees are shown in Figure 10.

Remark 1.9 The figure shows that as the ρ-value increases, the number of objects in Bnd(S) decreases, the number of objects in Neg(S) increases and the CER increases slightly. The number of objects in Bnd(S) is slightly higher when $\rho = 0.5$, we do not consider this value. $\rho = 0.9$, there are only 2 objects in Bnd(S) and the error rate is only 31.97%, so we finally choose the value of ρ as 0.9.

6.2 Experimental exploration of the utility pursuit coefficients

In the above part, we considered the effect of the dominance degree on decision outcomes. In this section, we consider the effect of utility pursuit coefficients ξ on ranking and categorisation results.

For the 244 objects in the Alnia fire dataset, we chose nine different utility pursuit coefficients in the range of 0

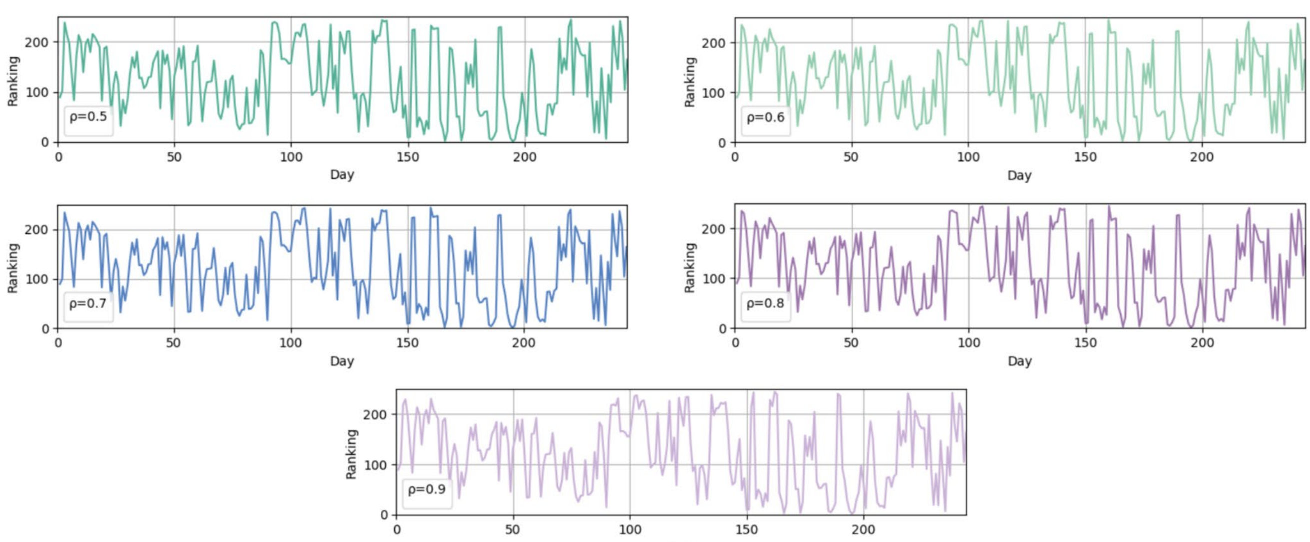

Fig. 9 Ranking under different dominance degree

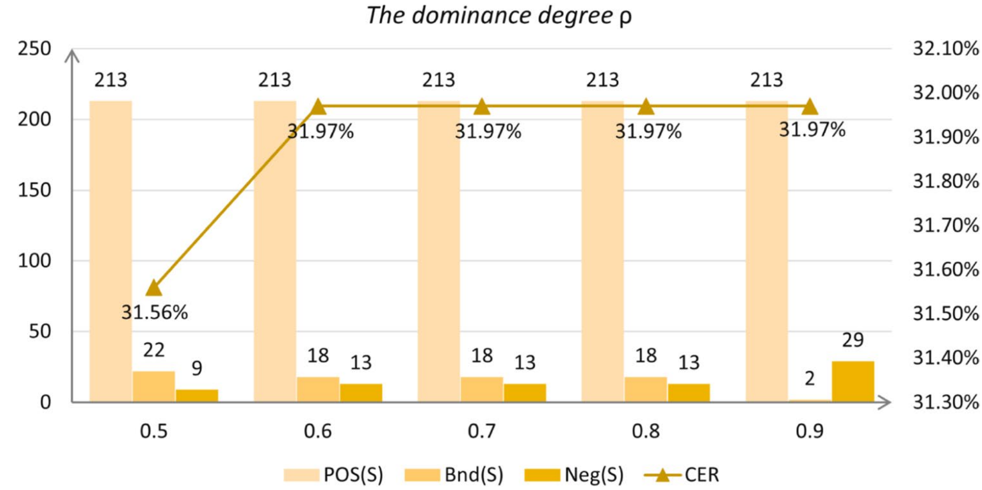

Fig. 10 Classification and CER under different dominance degree

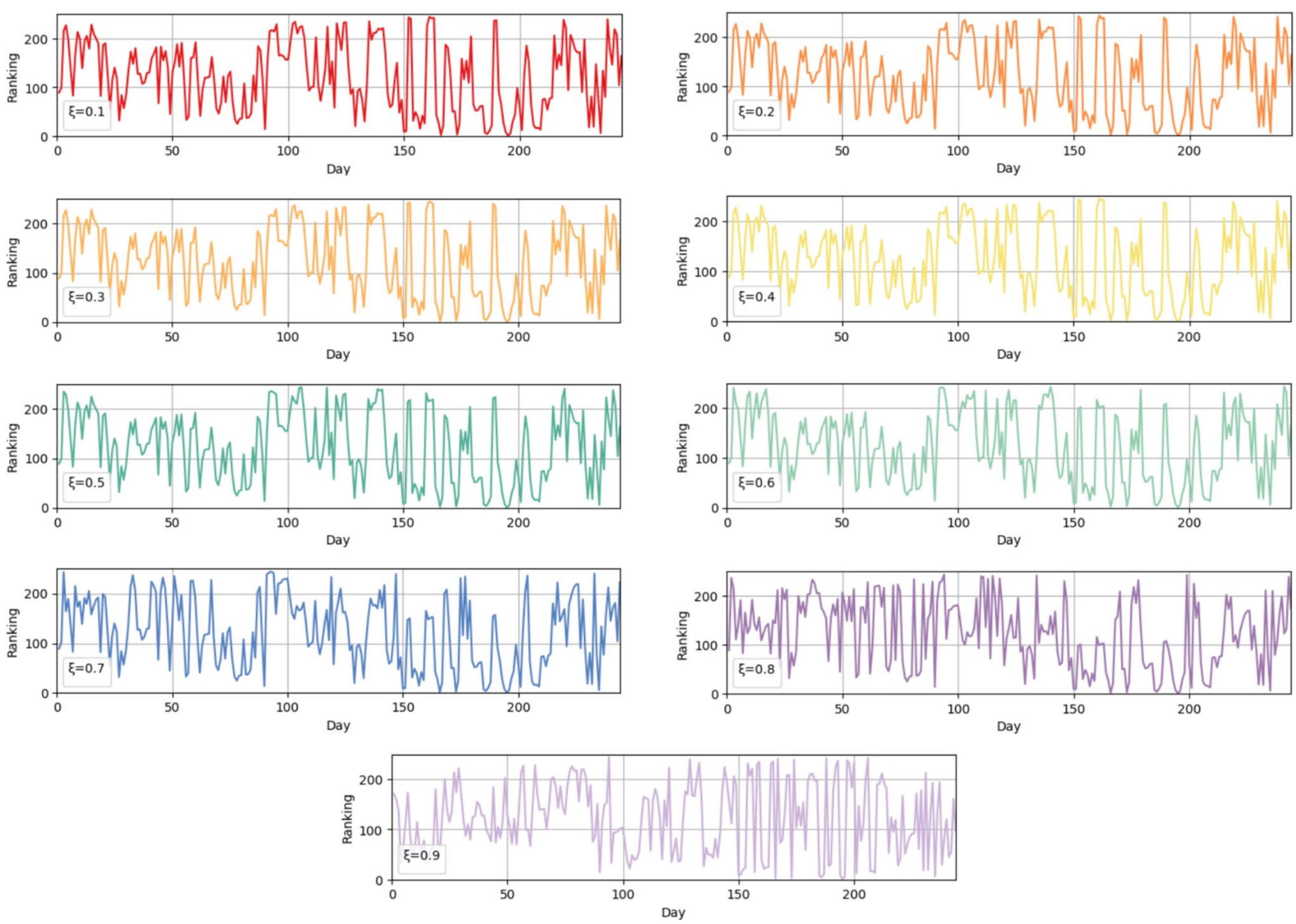

Fig. 11 Ranking under different the utility pursuit coefficients

to 1 with a step size of 0.1. Assume that $\rho = 0.9$; $\theta = 0.5$; $\delta = 0.7$. The ranking results corresponding to different utility pursuit coefficients are shown in Figure 11.

Remark 1.10 The nine subplots mentioned in the description above show that the sorting results remain largely consistent and unchanged for different values of ξ. This indicates that the method has good consistency and stability under different parameter values. This stability is essential for the reliability and credibility of the decision making process.

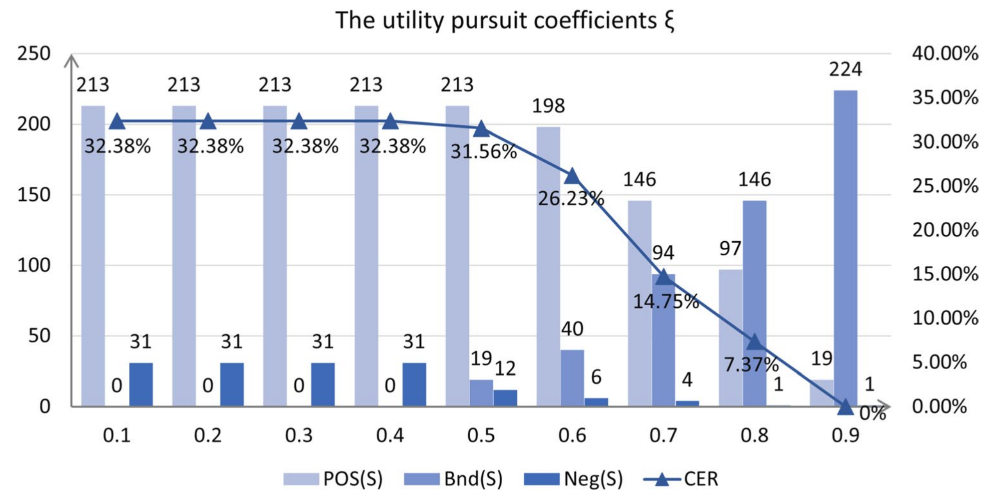

Fig. 12 Classification and CER under different utility pursuit coefficients

The classification results and CERs under different utility pursuit coefficients are shown in Figure 12.

Remark 1.11 When ξ is greater than 0.5, the number of objects in Bnd(S) increases dramatically as the utility pursuit coefficients increase. The objects are very hesitant and make more and more uncertain decisions, and although the CER decreases, the clustering index exceeds the reasonable range, so it is not taken into account; when ξ is less than 0.5, the decisions made by the objects are relatively certain. In order to make the model classification performance better, we did multiple sets of experiments (not shown here). It was finally determined that when the value of ξ is 0.417, there are only 2 objects in the model boundary domain, and the error rate is only 31.97%.

6.3 Experimental analysis on the risk aversion parameter

In this section we analyse the effect of the risk aversion coefficient θ on the decision outcome. Take the optimal dominance degree $\rho = 5$; the optimal utility pursuit coefficients $\xi = 0.417$, assuming $\delta = 0.7$. The risk aversion coefficient θ is varied from 0.1 to 0.9 with a step size of 0.1.

The ranking results corresponding to different risk aversion coefficients are shown in Figure 13.

Remark 1.12 As can be seen from the figure, the sorting results under different parameters have a high degree of similarity, i.e. the change of parameters does not have a large impact on the overall sorting results. Meanwhile, the optimal object in the Algerian forest fires dataset is still s_{195}, which shows that our proposed method meets the optimal consistency requirement of the multi-attribute decision problem.

The different classification results for different θ are shown in Figure 14.

Remark 1.13 In Figure14, with all other parameters unchanged, when $\theta < 0.4$, this is a binary classification result; when $\theta \geq 0.4$, as a increases, the number of objects in Bnd(S) gradually increases and the number of objects in Neg(S) gradually decreases, and no change in CER value after 0.5. This suggests that some objects that have been correctly categorised are now subject to uncertainty decisions. Based on the above considerations, we choose 0.5 as the optimal value of θ, because the model has the lowest error rate(31.97%) and the least number of uncertain objects(2 objects).

6.4 Experimental evaluation of the regret aversion parameter

In this section we analyse the regret aversion parameter. For now, the optimal parameter values obtained earlier are used for the other parameters: $\rho = 0.9$; $\xi = 0.417$; $\theta = 0.5$. We still use 0.1, 0.2, ..., 0.9 as the value of δ.

Remark 1.14 As shown in Figure 15, although the parameter changes altered the ranking to some extent, they did not affect the position of the best target (the 195th target). This indicates that the TWD-MADM-RT-HFS method aligns with the fundamental principle of multi-attribute decision-making problems, namely that the optimal solution remains stable under different values of the parameters and is not affected by the parameters.

The classification results and CERs under different regret avoidance values are shown in Figure 16.

Remark 1.15 As can be seen from the figure, the higher the regret aversion parameter δ, the more objects are in Bnd(S), indicating that the decision maker makes more uncertain decisions and misclassifies fewer objects to avoid his own regret. The CER reaches a minimum value of 31.56% at $\delta = 0.9$, but at this time the number of objects in the boundary region is high. Considering the accuracy of the model and its practicality in real problems, we choose a δ-value of 0.7. At this point, the number of uncertain decisions about the objects is lower, and 31.97% is only slightly lower than 31.56%, which makes the overall performance of the model better.

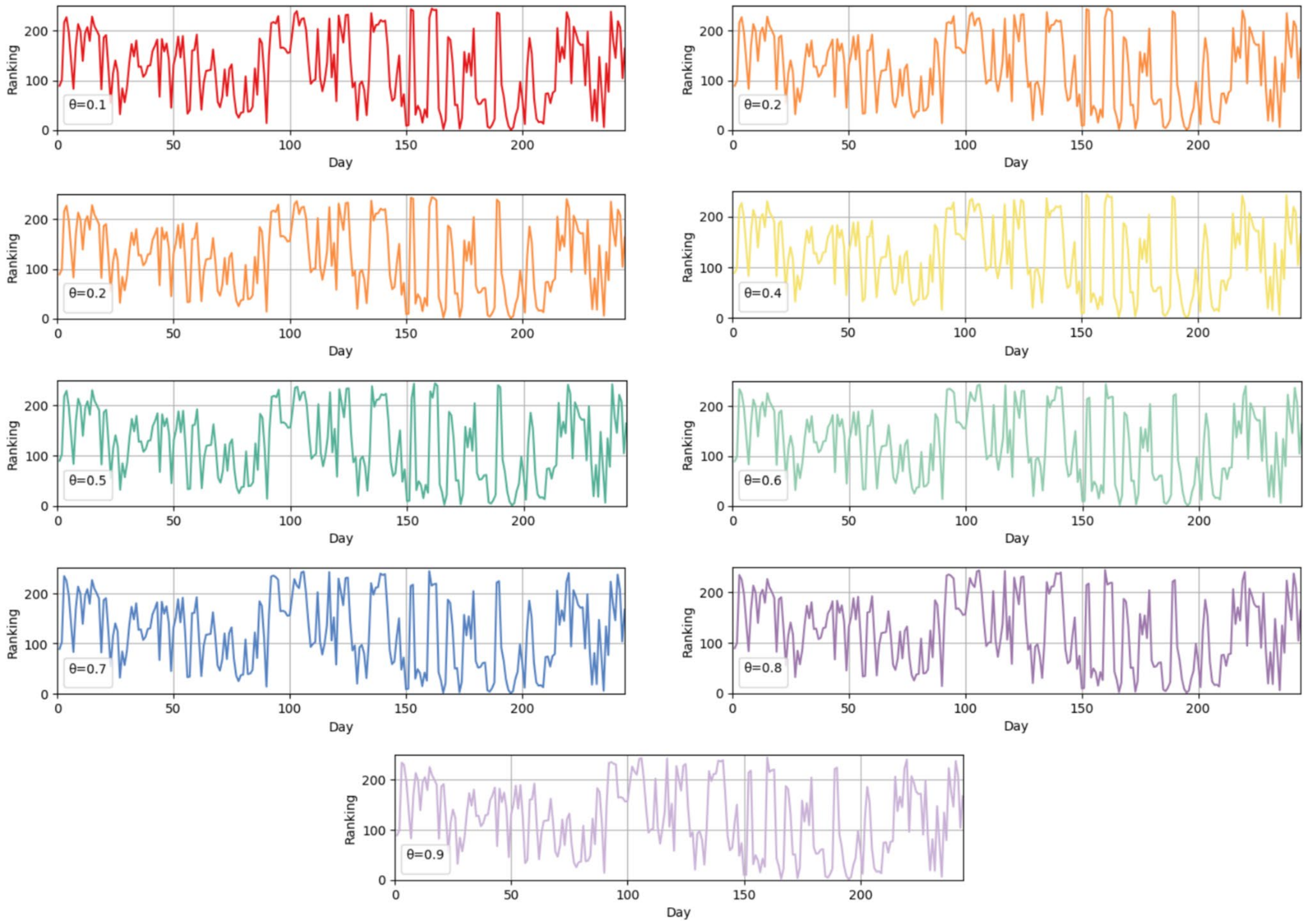

Fig. 13 Ranking under different risk aversion parameter

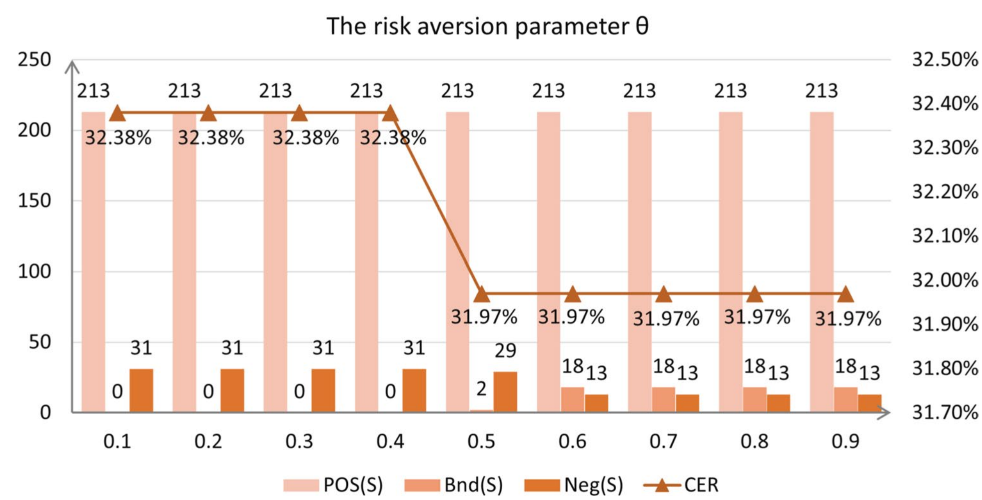

Fig. 14 Classification and CER under different risk aversion parameter

7 Conclusion

The RT method quantifies the decision maker's mental behavior when comparing multiple options. The HFS method provides an expression for uncertain information. The TWD method classifies objects into three categories. By combining the advantages of these three theories, we established the TWD-MADM-RT-HFS method. The main contributions of this paper are summarized below:

1. This paper combines TWD and RT for the first time in a hesitant fuzzy environment, considering the possibility

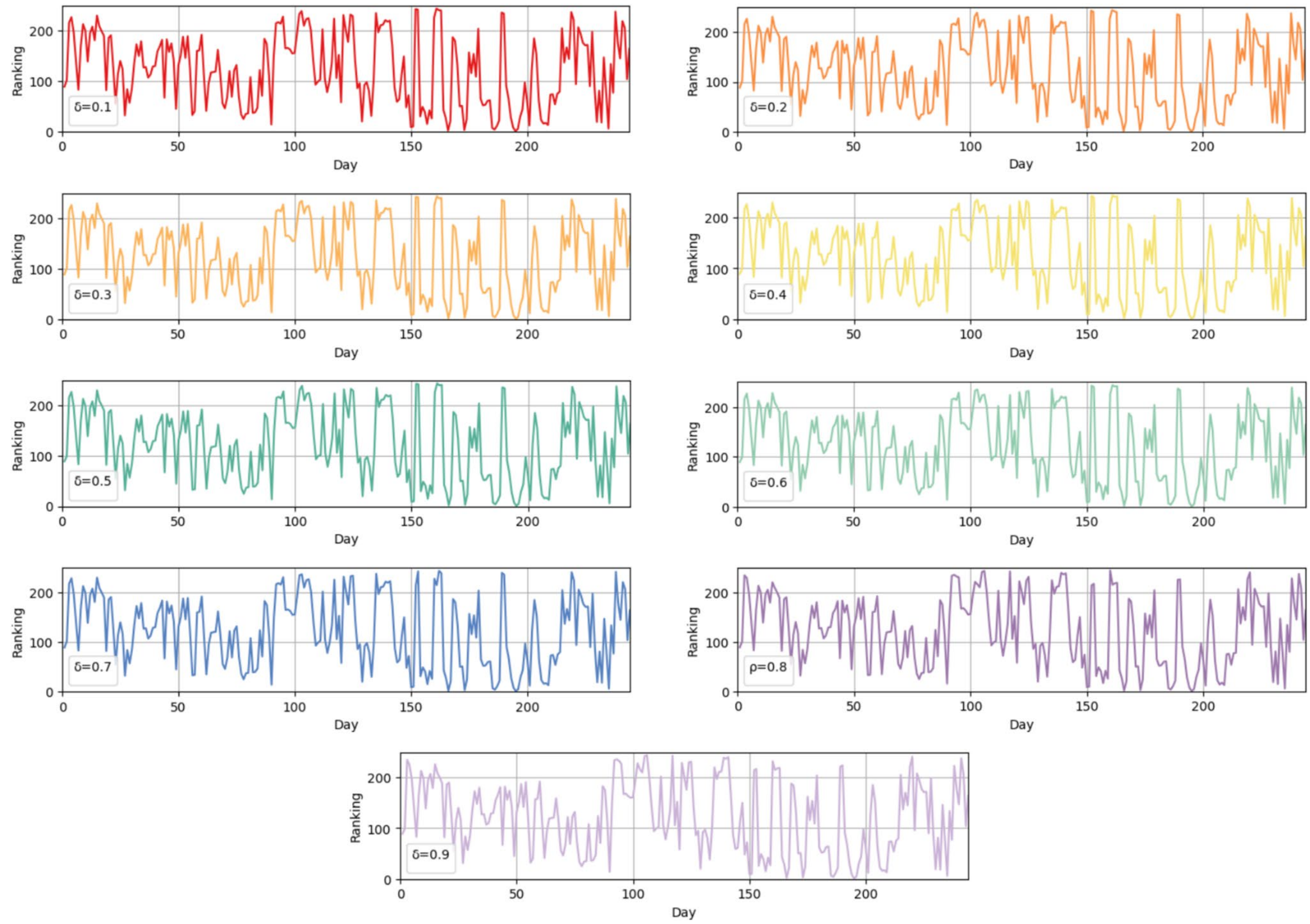

Fig. 15 Ranking under different regret aversion parameter

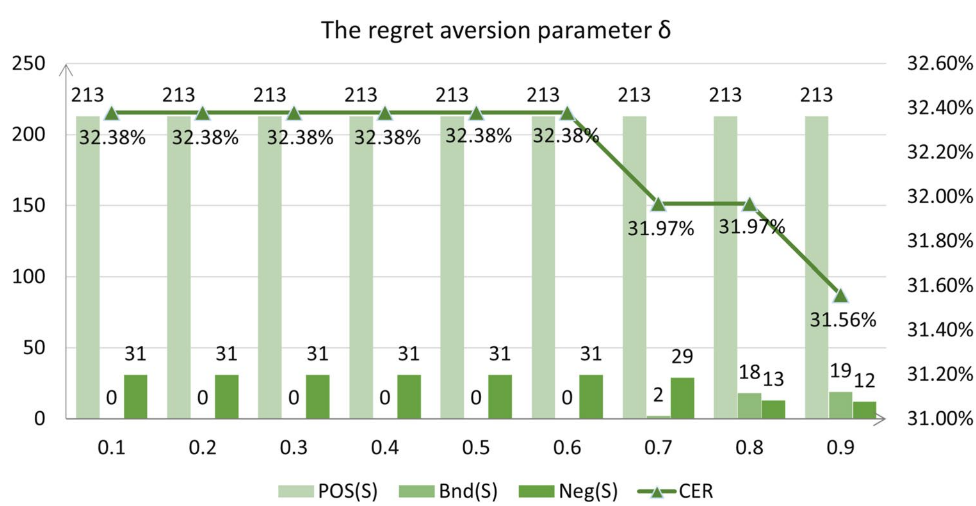

Fig. 16 Classification and CER under different regret aversion parameter

that decision makers may have emotions such as regret for their actions. This paper develops the TWD-MADM-RT-HFS method, which is capable of making decisions based on specific rules. This approach helps to achieve better results when dealing with uncertainty and emotional factors in the decision making process.

2. The constructed method employs the PROMETHEE-II method to calculate the gain/loss values and the satisfaction level to determine the weights. This approach ensures that the final expected utility function is highly objective, eliminating the dependence of the traditional TWD method on the loss function and conditional

probability. This implies that the TWD method is no longer constrained by the subjectively defined loss function and conditional probability in the decision-making process. Consequently, it can be more objective and flexible in decision-making, thereby enhancing its applicability and reliability.

3. This paper proposes a novel perceived utility function that always yields a non-negative result. This function enhances the interpretability and credibility of the model, facilitating a more comprehensive understanding of its output by decision makers. Furthermore, this function provides more effective decision support, simplifying the decision-making process and conferring numerous advantages and conveniences.
4. The proposed model is equipped with both ranking and classification functions. In the classification task, the model achieves both lower classification error rates and a reduction in the number of observed objects in the boundary domain compared to existing methods. This further confirms the effectiveness and accuracy of the model in practice. The model's comprehensive, flexible, and efficient design allows for the adaption of classification and ranking strategies according to the specific needs of the user, thereby rendering it applicable to a wide range of decision-making scenarios and requirements.

In light of the complexity of the real-world problem and the inadequacy of the existing models, it is recommended that future research consider the following directions:

1. This study presents a novel multi-attribute decision-making approach. In the future, the RT-based TWD model will be extended to the field of group decision making, further enriching and enhancing the decision theory in this field.
2. The attribute weights of the model proposed in this paper are assigned subjectively by experts based on their experience.In the future, we intend to use an objective method to determine the weights.
3. In the context of practical matters, the phenomenon of missing data is a common occurrence. In the future, it would be beneficial to propose methods for incomplete information systems that address the impact of missing data on decision models.

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Declarations

Competing interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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