Dynamic Multi-View Classification and Knowledge Fusion: A Fuzzy Concept-Cognitive Learning Perspective

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***Abstract*—Recently, multi-view data have grown significantly in practical scenarios. Compared with single-view data, they can comprehensively describe objects through diverse types of features. However, their inherent heterogeneity introduces new challenges for knowledge discovery, especially in dynamic environments. To effectively represent knowledge in dynamic multi-view data, this paper proposes a dynamic multi-view concept-cognitive learning (DMVCCL) model. First, a multi-view knowledge representation framework is established, which uses fuzzy three-way concepts as basic carriers. The natural hierarchical relationship between concepts is utilized to precisely represent knowledge in multi-view data. Then, for dynamic multi-view data, a clue-based dynamic concept updating mechanism is designed. This mechanism leverages the varying sensitivities of concepts at different granularity levels to data changes, enabling learning concepts at the optimal granularity level. Moreover, the weights of each view are assigned based on the representation capability of the learned concepts, and a multi-view classification method is designed using the similarity between concepts and data. Finally, a series of comparative experiments are conducted to verify the effectiveness of the proposed method.**

***Index Terms*—Concept-cognitive learning, fuzzy concept, granular computing, knowledge discovery, multi-view classification.**

I. Introduction

With the advancement of information technology, many industries have generated complex multi-view data, which can describe objects from multiple views. For example, a scene can be represented by multiple views such as visual images, audio recordings, and textual descriptions. Notice that humans can naturally process multi-view data, enabling a comprehensive understanding by integrating visual, tactile, and other sensory inputs. Consequently, researching computational paradigms for multi-view data is of significant importance, as it enables models to understand objects more comprehensively. This reality has driven the emergence and development of multi-view learning [1], [2].

Broadly speaking, multi-view data encompasses not only information represented through different media but also data obtained through various descriptive methods [3], [4]. For example, an image can be described by scale-invariant features, and it can also be depicted by texture features. After obtaining multi-view data, a key issue is how to integrate data from multiple views to make a prediction. Many researchers have conducted studies on this issue. For example, by considering the correlations and higher-order information among features within each view, Liang et al. proposed an intra-view feature fusion method to enhance multi-view classification performance [3]. Zhang et al. [5] introduced a metric learning method that improves classification performance by considering both intra-view class separability and inter-view correlation. Hu et al. [6] characterized the distribution of multi-view data using anchor points, establishing a multi-view fuzzy classification model. Han et al. [7], [8] investigated trusted multi-view classification methods by assessing the uncertainty of each view. Additionally, several studies have explored multi-view fusion methods that incorporate deep learning techniques and have been used in fields such as image fusion [9], [10] and epilepsy electroencephalographic recognition [11], [12].

It is noteworthy that humans can flexibly process multi-view data, abstracting essential features from different views and analyzing phenomena at varying levels of granularity [13]–[15] to achieve a comprehensive understanding of objects. Therefore, processing multi-view data by simulating the way humans cognize things is of significant importance. One prominent characteristic of human cognition is the ability to abstract the common features of objects, and represents them in the form of concepts [16]. The fundamental elements of a concept are its extent and intent. The extent refers to a set of objects, while the intent is the set of the common features of these objects. For example, for the concept "prime numbers", the intent is defined as natural numbers that have no divisors other than 1 and themselves, while the extent is the set of all prime numbers $\{2, 3, 5, 7, \ldots\}$. Inspired by the concept in philosophy, Wille [17] first proposed a formal definition of concepts for extracting valuable information from tabular data. Subsequently, formal concept analysis theory (FCA) was developed. In FCA, the extent of a concept is defined as a set of

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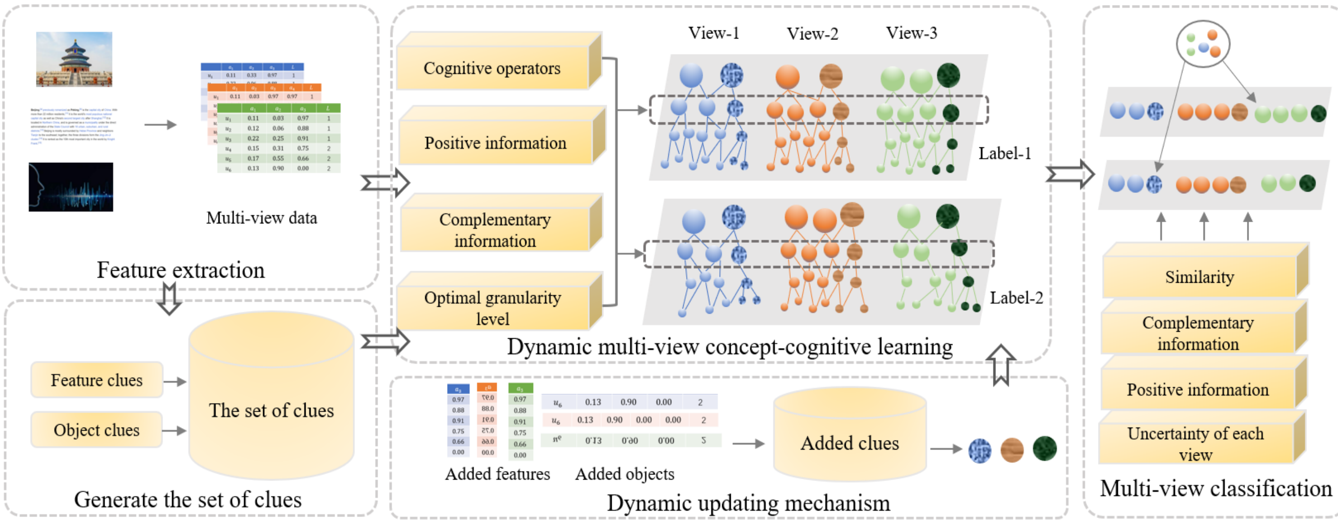

Fig. 1. Procedure of fuzzy concept-cognitive learning model for dynamic multi-view classification

objects, whereas the intent is constituted by the set of features that are shared among these objects. The main tool of FCA is the concept lattice, which is a partially ordered structure formed by concepts. Although concept lattice can reflect the relationship between objects and features comprehensively, generating a concept lattice is extremely time-consuming. Therefore, some studies attempted to combine machine learning methods and granular computing theory to efficiently learn concepts from data. These studies have given rise to a novel knowledge representation paradigm, known as concept-cognitive learning (CCL) [18]–[21]. CCL aims to explore a more efficient and precise method for discovering knowledge embedded in data through concepts as carriers. A series of CCL models were developed subsequently. On one hand, some studies investigated mechanisms for quickly learning concepts from data. For example, Xu et al. explored concept learning mechanisms from a progressive cognition perspective [22] and a movement perspective [23]. Zhang et al. [24] used variable precision cognitive operators to learn concepts from fuzzy data. On the other hand, some studies utilized concepts as knowledge carriers for various tasks. For instance, Yuan et al. [25] created a dynamic classification model using fuzzy three-way concepts. Guo et al. [26] introduced a memory-based CCL model focused on big concepts for dynamic knowledge fusion. The dynamic updating mechanisms of concepts were designed in [25] and [26], which both emphasized learning concept at different granularity levels to capture knowledge from data. These mechanisms also demonstrated good performance in dynamic environments. One major reason is that concepts learned at different granularity levels have varying degrees of sensitivity to data changes, and concepts with stronger representation capabilities can be obtained by searching for the optimal granularity level. Besides, Ding et al. [27] defined the interval fuzzy concept for classification. Wang et al. [28] proposed a multi-view knowledge representation framework based on the idea of CCL. Deng et al. [29] used concepts as carriers for hierarchical classification. Wu et al. [30], [31] developed CCL models for multi-label classification . Zhou et al. [32], [33] investigated CCL models for skill assessment by combining knowledge space theory. Furthermore, some CCL models were developed for clique identification [34] and knowledge reasoning [35].

By summarizing existing CCL models, several advantages of using concepts as carriers for knowledge representation can be identified:

1) Interpretability. Concepts are the fundamental units of human cognition in philosophy [19]. Therefore, using concepts as carriers of knowledge aligns with the way humans cognize objects. Besides, the development of FCA theory provides a solid mathematical foundation for knowledge discovery with concepts as knowledge carriers.
2) Flexibility. Concepts are learned through cognitive operators, allowing flexible definitions for various situations. Existing research indicates that CCL models can effectively address knowledge representation challenges in dynamic environments [25], incomplete scenarios [36], and other contexts.
3) Multi-granularity characteristic. There exists a specialization-generalization relationship between concepts. The generalized concept reflects the common features, while the specialized concept reflects the unique features. Thus, concepts can simultaneously convey both common and unique features of objects, which naturally stimulates the multi-granularity characteristic of human thought.

Based on the above analysis, it can be observed that: On the one hand, existing CCL models focus on knowledge discovery from single-view data, and have demonstrated excellent classification results for various data types and in various environments. However, few studies have focused on developing CCL models for multi-view data. A key feature of CCL is its emphasis on discovering and integrating knowledge in data through a manner that simulates human cognition. Intuitively, humans naturally cognize things from multiple views. From this perspective, when CCL models can handle multi-view data can they obtain higher cognitive levels. On the other hand, although existing multi-view learning methods achieved remark performance on numerous tasks, these methods lack interpretability, and seldom address dynamic multi-view data systematically. Based on previous discussions, CCL has strong interpretability and can flexibly handle dynamic data. Therefore, exploring CCL models for dynamic multi-view data is a meaningful research topic. Based on these observations, this paper proposes a dynamic multi-view concept-cognitive learning model, and Fig. 1 describes the overall procedure. The specific innovations are as follows:

1) It proposes a framework for effectively representing the knowledge in multi-view data in the form of fuzzy three-way concepts.
2) It leverages the hierarchical relationships among concepts to accurately characterize the common and unique features of objects.
3) It utilizes extent information to update concepts at the optimal granularity level to maintain the representational capacity of concepts.
4) It designs a multi-view classification method, by considering the representational capability of concepts across different views and the similarity between objects and concepts.

The arrangement of this paper is as follows. The next section introduces some basic definitions about concepts. Section III discusses the proposed method in detail. Section IV presents and analyzes the experimental results. The conclusions are presented in Section V.

II. PRELIMINARIES

This section provides some basic notations with respect to fuzzy concepts. More detailed illustrations can be found in [25], [37], [38].

A. Fuzzy concepts

Formally, a tuple $< U, A, C >$ is known as a dataset, where

1) $U = \{u_1, u_2, \ldots, u_M\}$ is the set of objects;
2) $A = \{a_1, a_2, \ldots, a_N\}$ is the set of features;
3) $C : U \times A \to [0, 1]$ is a mapping that reflects the values of objects under features.

For any $u \in U$ and $a \in A$, $C(u, a)$ is the value of u under feature a. This paper assumes that $C(u, a) \in [0, 1]$, or can be scaled to fall within $[0, 1]$. $C(u, a)$ can be understood as the degree to which object u possesses feature a. The larger $C(u, a)$ is, the higher the degree of possession of a by u.

To extract valuable information from data, some studies have proposed methods for representing knowledge in data through fuzzy concepts. Generally, a fuzzy concept is a pair (E, I), where $E \subseteq U$ denotes the extent, reflecting object information, and I represents the intent, reflecting feature information. According to fuzzy set theory [39], $I = \{I(a_1), I(a_2), \ldots, I(a_N)\}$ can be regarded as a fuzzy set on A. For any $a_n \in A$, $I(a_n) \in [0, 1]$ denotes the membership degree of a_n to I. The set of all fuzzy sets on A is denoted as $2^{\widetilde{A}}$, and the power set of U is represented by 2^U. For any $I_1, I_2 \in 2^{\widetilde{A}}$, the intersection operation $\cap$ and the union operation $\cup$ between I_1 and I_2 are defined as follows. If $I = I_1 \cap I_2$, then $I(a_n) = \min\{I_1(a_n), I_2(a_n)\}$, and if $I = I_1 \cup I_2$, then $I(a_n) = \max\{I_1(a_n), I_2(a_n)\}$.

To learn fuzzy concepts from data, a pair of cognitive operators $\mathcal{L} : 2^U \to 2^{\widetilde{A}}$ and $\mathcal{F} : 2^{\widetilde{A}} \to 2^U$ can be defined.

Definition 1. *For a dataset $< U, A, C >$, suppose $E \in 2^U$ and $I \in 2^{\widetilde{A}}$, a pair of operators $\mathcal{L}$ and $\mathcal{F}$ is defined as follows:*

$$\begin{aligned} \mathcal{L}(E)(a) &= \bigwedge_{u \in E} C(u, a), a \in A, \\ \mathcal{F}(I) &= \{u \in U | I(a) \leq C(u, a), \forall a \in A\}, \end{aligned} \tag{1}$$

where $\bigwedge$ represents the minimum operation.

For any $E \in 2^U$ and $I \in 2^{\widetilde{A}}$, (E, I) is referred to as a fuzzy concept if $\mathcal{L}(E) = I$ and $\mathcal{F}(I) = E$. Intuitively, I can be understood as the features jointly possessed by the objects in E.

B. Fuzzy three-way concepts

To enable a comprehensive representation of knowledge, the definition of three-way concepts is proposed [40]. The intent of a three-way concept consists of two parts. The first part reflects the set of features that are jointly possessed by the objects in the extent, while the second part reflects the set of features that are jointly not possessed. In this way, the relationship between objects and features can be fully represented. Besides, Yuan et al. [25] further investigated fuzzy three-way concepts for dynamic knowledge discovery.

For a dataset $< U, A, C >$, let $< U, A, C^- >$ be the complement of $< U, A, C >$, where $C^-(u, a) = 1 - C(u, a)$ for any $u \in U$ and $a \in A$. It can be observed that $C^-(u, a)$ reflects the degree to which u does not possess a. Then, a pair of negative cognitive operators can be defined to learn the complementary information.

Definition 2. *For $E \in 2^U$ and $I^- \in 2^{\widetilde{A}}$, a pair of negative operator $\mathcal{L}^-$ and $\mathcal{F}^-$ is defined as follows:*

$$\begin{aligned} \mathcal{L}^-(E)(a) &= \bigwedge_{u \in E} C^-(u, a), a \in A, \\ \mathcal{F}^-(I^-) &= \{u \in U | I^-(a) \leq C^-(u, a), \forall a \in A\}. \end{aligned} \tag{2}$$

The definition of positive operators have been given Definition 1. Then, a pair of three-way cognitive operators is generated to learn fuzzy three-way concepts.

Definition 3. *For $E \in U$ and $I^+, I^- \in 2^{\widetilde{A}}$, a pair of three-way cognitive operators $\mathcal{L}^*$ and $\mathcal{F}^*$ is defined as follows:*

$$\begin{aligned} \mathcal{L}^*(E) &= \big(\mathcal{L}(E), \mathcal{L}^-(E)\big), \\ \mathcal{F}^*(I^+, I^-) &= \mathcal{F}(I^+) \cap \mathcal{F}^-(I^-). \end{aligned} \tag{3}$$

According to Definition 3, $(E, (I^+, I^-))$ is referred to as a fuzzy three-way concept if

$$\mathcal{L}^*(E) = (I^+, I^-), \mathcal{F}^*(I^+, I^-) = E. \tag{4}$$

It is noteworthy that fuzzy three-way concepts can simultaneously characterize the degree to which the features are possessed and not possessed by the objects in E. In the following, fuzzy three-way concepts will be utilized for knowledge representation of multi-view data. Without causing confusion, we will refer to the fuzzy three-way concepts simply as concepts in the subsequent discussion.

There exists a partial order relationship $\preceq$ among concepts, defined as follows:

$$(E_1,(I_1^+,I_1^-)) \preceq (E_2,(I_2^+,I_2^-)) \Leftrightarrow E_1 \subseteq E_2. \tag{5}$$

$(E_1,(I_1^+,I_1^-))$ is referred to as a specialized concept of $(E_2,(I_2^+,I_2^-))$, and $(E_2,(I_2^+,I_2^-))$ is called a generalized concept of $(E_1,(I_1^+,I_1^-))$. The extent of a generalized concept includes more objects, and therefore its intent reflects the common features of these objects. In contrast, the extent of a specialized concept includes fewer objects, and thus its intent reflects the specific features of a relatively smaller group of objects.

III. The Proposed Method

This section discusses in detail the proposed dynamic multi-view concept-cognitive learning model. It consists of three parts: the construction of multi-view concept spaces, concept-based multi-view classification, and the dynamic updating mechanism of concepts. These parts correspond to Subsections III-A, III-B, and III-C, respectively.

A. Construction of multi-view concept spaces

First, a multi-view dataset $\mathcal{D}$ with P views is composed of multiple single-view datasets, i.e., $\mathcal{D}=\{\mathcal{D}_1,\mathcal{D}_2,\dots,\mathcal{D}_P\}$. For any $\mathcal{D}_p\in\mathcal{D}$, $\mathcal{D}_p=<U,A_p,C_p,L>$, where

1) $U=\{u_1,u_2,\dots,u_M\}$ is the set of objects;
2) $A_p=\{a_1,a_2,\dots,a_{N_p}\}$ is the set of features under the p-th view, and N_p is the number of features under the p-th view;
3) $C_p: U\times A_p\to[0,1]$ is a mapping, and $C_p(u,a)$ denotes the value of object u on feature a;
4) $L=\{l_1,l_2,\dots,l_K\}$ is the set of labels, where each object only has one label. It forms a partition $\{U_1,U_2,\dots,U_K\}$ on U, where $U_k(k=1,2,\dots,K)$ is the set of objects with label l_k.

For convenience, we divide the multi-view dataset into $K\times P$ parts based on view and label information, denoted as $\{D_p^k|p=1,2,\dots,P,k=1,2,\dots,K\}$. For any $p\in\{1,2,\dots,P\}$ and $k\in\{1,2,\dots,K\}$, $D_p^k=<U_k,A_p,C_p^k>$ is the data subset associated with label l_k under the p-th view. For any $u\in U_k$ and $a\in A_p$, $C_p^k(u,a)$ is the value of u on feature a under the p-th view. Then, we attempt to learn concepts from D_p^k to represent the knowledge associated with label l_k under the p-th view.

A characteristic of CCL is that concepts can be learned from clues. These clues can be either a set of objects or a fuzzy set on A_p. In this paper, the set of object clues Obj_p^k and the set of feature clues Fea_p^k are defined as follows, respectively.

$$\begin{aligned} &Obj_p^k=\{\{u\}|u\in U_k\},\\ &Fea_p^k=\bigcup_{a\in A_p}\{\{u\in U_k|C_p^k(u,a)>\frac{\max(C_p^k(:,a))}{2}\}\}. \end{aligned} \tag{6}$$

Here, $\max(C_p^k(:,a))$ represents the maximum value under feature a in the data subset D_p^k. Then, the set of clues with respect to D_p^k is defined as $Clue_p^k=Obj_p^k\cup Fea_p^k$. If we restrict the operators in Definition 3 on D_p^k, a pair of operators $\mathcal{L}_p^{k*}$ and $\mathcal{F}_p^{k*}$ can be defined to learn concepts from D_p^k.

Definition 4. *Let $D_p^k=<U_k,A_p,C_p^k>$ be the data subset associated with label l_k under the p-th view, for any $E\in 2^{U_k}$ and $I^+,I^-\in 2^{\widetilde{A}_p}$, four operators can be define as follows:*

$$\begin{aligned} \mathcal{L}_p^k(E)(a)&=\bigwedge_{u\in E}C_p^k(u,a),a\in A_p,\\ \mathcal{F}_p^k(I^+)&=\{u\in U_k|I^+(a)\le C_p^k(u,a),\forall a\in A_p\},\\ \mathcal{L}_p^{k-}(E)(a)&=\bigwedge_{u\in E}C_p^{k-}(u,a),a\in A_p,\\ \mathcal{F}_p^{k-}(I^-)&=\{u\in U_k|I^-(a)\le C_p^{k-}(u,a),\forall a\in A_p\}, \end{aligned} \tag{7}$$

where $C_p^{k-}(u,a)=1-C_p^k(u,a)$ for any $a\in A_p$ and $u\in U_k$. Based on Eq. (7), the three-way cognitive operators $\mathcal{L}_p^{k*}$ and $\mathcal{F}_p^{k*}$ are defined as follows:

$$\begin{aligned} \mathcal{L}_p^{k*}(E)&=\big(\mathcal{L}_p^k(E),\mathcal{L}_p^{k-}(E)\big),\\ \mathcal{F}_p^{k*}(I^+,I^-)&=\mathcal{F}_p^k(I^+)\cap\mathcal{F}_p^{k-}(I^-). \end{aligned} \tag{8}$$

To facilitate understanding of how to learn concepts from clues, the following properties are provided, which have been proved in [25].

Property 1. *For any $E,E_1,E_2\in Clue_p^k$, we have:*

1) $E\subseteq\mathcal{F}_p^{k*}\circ\mathcal{L}_p^{k*}(E)$;
2) $E_1\subseteq E_2\Leftrightarrow\mathcal{F}_p^{k*}\circ\mathcal{L}_p^{k*}(E_1)\subseteq\mathcal{F}_p^{k*}\circ\mathcal{L}_p^{k*}(E_2)$;
3) $\mathcal{L}_p^{k*}(E)=\mathcal{L}_p^{k*}\circ\mathcal{F}_p^{k*}\circ\mathcal{L}_p^{k*}(E)$;
4) $\mathcal{L}_p^k(E_1\cup E_2)=\mathcal{L}_p^k(E_1)\cap\mathcal{L}_p^k(E_2)$;
5) $\mathcal{L}_p^{k^-}(E_1\cup E_2)=\mathcal{L}_p^{k^-}(E_1)\cap\mathcal{L}_p^{k^-}(E_2)$.

Property 1 indicates that starting from clues, concepts can be learned by $\mathcal{L}_p^{k*}$ and $\mathcal{F}_p^{k*}$. Specifically, for any $E\in Clue_p^k$, $\big(\mathcal{F}_p^{k*}\circ\mathcal{L}_p^{k*}(E),\mathcal{L}_p^{k*}(E)\big)$ is a concept. Intuitively, concepts learned based on clues from Obj_p^k tend to be specialized concepts, meaning they include fewer objects in their extents, and their intents reflect the unique features of a relatively small group of objects. Besides, the set of feature-generating clues are also introduced to further enhance the richness of the generated concepts. In the following, we consider using concepts as carriers to progressively refine the knowledge with respect to D_p^k. First, the initial concept subspace GCS_p^k is defined as follows:

$$GCS_p^k=\big\{\big(\mathcal{F}_p^{k*}\circ\mathcal{L}_p^{k*}(E),\mathcal{L}_p^{k*}(E)\big)|E\in Clue_p^k\big\}. \tag{9}$$

Based on GCS_p^k, we can initially learn the concepts associated with label l_k under the p-th view. And define $GCS=$

$\{GCS_p^k|p = 1,2,\dots,P, k = 1,2,\dots,K\}$ as the initial concept space.

Then, we aim to learn concepts at different granularity levels by continuously merging similar concepts. The following equations are defined to measure the distance between two concepts. Suppose $Cpt_1 = (E_1,(I_1^+,I_1^-))$ and $Cpt_2 = (E_2,(I_2^+,I_2^-))$ are two concepts in GCS_p^k. Then, the extent distance between them is defined as

$$Edis(Cpt_1,Cpt_2) = 1 - \frac{|E_1 \cap E_2|}{|E_1 \cup E_2|}, \tag{10}$$

where $|\cdot|$ represents the cardinality of a set. The intent distance between them is defined as

$$Idis(Cpt_1,Cpt_2) = \frac{||I_1^+ - I_2^+||_2 + ||I_1^- - I_2^-||_2}{2N_p}, \tag{11}$$

where $||\cdot||_2$ represents 2-norm. Based on Eqs. (10) and (11), the total distance between Cpt_1 and Cpt_2 is defined as

$$Tdis(Cpt_1,Cpt_2) = \frac{Edis(Cpt_1,Cpt_2) + Idis(Cpt_1,Cpt_2)}{2}. \tag{12}$$

The smaller the value of $Tdis(Cpt_1,Cpt_2)$, the higher the similarity between Cpt_1 and Cpt_2.

Here, a parameter i is introduced, which represents the number of concepts that are similar to a given concept. More specifically, given a concept $(E,(I^+,I^-)) \in GCS_p^k$, the distances between $(E,(I^+,I^-))$ and each concept in GCS_p^k are calculated using Eq. (12), where a smaller distance indicates a higher similarity to $(E,(I^+,I^-))$. Suppose $(E_1,(I_1^+,I_1^-)),(E_2,(I_2^+,I_2^-)),\dots,(E_i,(I_i^+,I_i^-))$ are the top-i concepts that are most similar to $(E,(I^+,I^-))$. Intuitively, the concepts in GCS_p^k are specialized concepts, reflecting unique features of the objects. By combining the extents of these concepts as clues, we can learn a more generalized concept. This process is described as follows,

$$\Big(\mathcal{F}_p^{k*} \circ \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j), \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j)\Big). \tag{13}$$

For each concept in GCS_p^k, a fused concept can be obtained according to Eq. (13) and parameter i. The updated concept subspace $CS_p^{k,i}$ is then generated, i.e.,

$$CS_p^{k,i} = \Big\{\Big(\mathcal{F}_p^{k*} \circ \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j), \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j)\Big) | \; (\mathcal{F}_p^{k*} \circ \mathcal{L}_p^{k*}(E), \mathcal{L}_p^{k*}(E)) \in GCS_p^k\Big\}. \tag{14}$$

$CS_p^{k,i}$ is called the concept subspace under the granularity level i. And the learned multi-view concept space is denoted as $CS = \{CS_p^{k,i}|p = 1,2,\dots,P, k = 1,2,\dots,K\}$.

Property 2. *For any $(E,(I^+,I^-)) \in CS_p^{k,i}$, there exists $(E',(I'^+,I'^-)) \in CS_p^{k,i+1}$, such that:*

1) $E \subseteq E'$;
2) $I'^+ \subseteq I^+$;
3) $I'^- \subseteq I^-$.

Proof.
- For 1), according to Eq. (13), we know that there exists $\{(E_1,(I_1^+,I_1^-)),(E_2,(I_2^+,I_2^-)),\dots,(E_{i+1},(I_{i+1}^+,I_{i+1}^-))\} \subseteq GCS_p^k$ such that $\mathcal{F}_p^{k*} \circ \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j) = E$ and $\mathcal{F}_p^{k*} \circ \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i+1} E_j) = E'$. Since $\bigcup_{j=1}^{i} E_j \subseteq \bigcup_{j=1}^{i+1} E_j$, according to Property 1. 2), it follows that $E \subseteq E'$.
- 2) and 3) can be proved based on 1) and Property 1.

□

According to Property 2, as i is continuously increased, the number of objects in the extents of the generated concepts grows larger, and the intents increasingly reflect the common features of these objects. To find a balance between the generalization and specialization of concepts, it is necessary to select the optimal granularity level i. This procedure is illustrated in Fig. 2, which is similar to the process that humans perceive objects at different levels of granularity, seeking a balance between common and unique features to achieve a comprehensive understanding of objects.

Based on the above discussion, the algorithm to learn a multi-view concept space is presented in Algorithm 1, and the time complexity is $O(\sum_{p=1}^{P}\sum_{k=1}^{K}|U_k|^2|A_p| + |U_k||A_p|^2)$. It is noted that the set of extents of all concepts in $CS_p^{k,i}$ forms a covering on U_k, i.e., the information provided by all objects are considered when constructing a multi-view concept space.

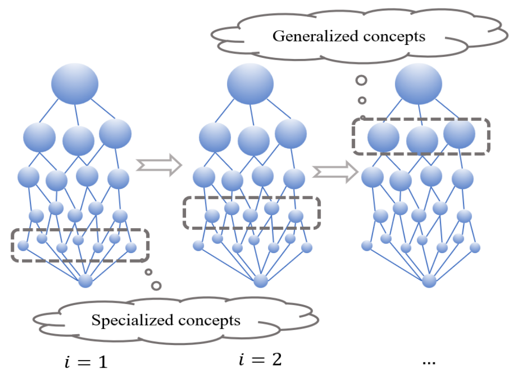

Fig. 2. Concepts generated at different granularity levels

B. Multi-view classification via concepts

After obtaining a multi-view concept space CS, this subsection utilizes the concepts in CS for multi-view classification.

Assume that u is a newly added object with $A_p(u) = \{a(u)|a \in A_p\}(p \in \{1,2,\dots,P\})$ as the corresponding feature values of the p-th view. Define $A_p^-(u) = \{1-a(u)|a \in A_p\}$ as the complementary information. Then, a $K \times P$-dimensional matrix T is used to represent the nearest distance between u and each concept subspace, where

$$T(k,p) = \min_{(E,(I^+,I^-)) \in CS_p^{k,i}} \{||A_p(u)-I^+||_2 + ||A_p^-(u)-I^-||_2\}. \tag{15}$$

Algorithm 1 Construction of multi-view concept spaces

Input: A multi-view dataset $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, \ldots, \mathcal{D}_P\}$, the granularity level i;
Output: A multi-view concept space CS.
Dividing $\mathcal{D}$ into $\{D_p^k | p = 1, 2, \ldots, P, k = 1, 2, \ldots, K\}$;
for $p \in \{1, 2, \ldots, P\}$ **do**
  for $k \in \{1, 2, \ldots, K\}$ **do**
    Calculate the set of clues $Clue_p^k$ based on Eq. (6);
    Calculate the initial concept subspace GCS_p^k according to Eq. (9);
    for each $(E, (I^+, I^-)) \in GCS_p^k$ **do**
      Compute the top-i concept that are similar to $(E, (I^+, I^-))$ in GCS_p^k according to Eq. (12);
      Obtain the fused concept according to Eq. (13) and add it to $CS_p^{k,i}$;
    end for
  end for
end for
Reture the multi-view concept space $CS = \{CS_p^{k,i} | p = 1, 2, \ldots, P, k = 1, 2, \ldots, K\}$.

The maximum value in the p-th column of T is denoted as $\max(T(:, p))$. Based on T, we can then compute the similarity matrix S between u and each concept subspace as

$$S(k, p) = 1 - \frac{T(k, p)}{\max(T(:, p))}. \tag{16}$$

Intuitively, a larger value of $S(k, p)$ indicates a stronger association between object u and the concept subspace $CS_p^{k,i}$ under the p-th view, i.e., a higher probability that the label of u is l_k. Based on S, the label of u is given by the following equation:

$$l_k = \arg\max_k \sum_{p=1}^{P} W(p) S(k, p), \tag{17}$$

l_k is the label of u, W is a weight vector and $W(p) \in [0, 1]$. In order to obtain W, we first select 70% of the objects from the training set for generating a multi-view concept space, while the remaining 30% of the objects from the training set are used as the test set. According to Eq. (17), the similarity matrix S of an object u of the test set can be calculated. The label of u under the p-th view is given by the following equation:

$$l_k = \arg\max_k S(k, p). \tag{18}$$

By comparing the predicted labels with the ground-true labels, the number of correctly predicted objects $Z(p)$ for the p-view is calculated. Then, W can be calculated based on the following equations:

$$\begin{aligned} Temp(p) &= \exp\left(\frac{Z(p) - \max(Z)}{\sigma}\right), \\ W(p) &= \frac{Temp(p)}{\sum_{p=1}^{P} Temp(p)}, \end{aligned} \tag{19}$$

Algorithm 2 Multi-view classification via concepts

Input: A multi-view concept space CS, new object with multi-view features $A_p(u) = \{a(u) | a \in A_p\} (p = 1, 2, \ldots, P)$, the weight vector W;
Output: The label l_k of u.
for $p \in \{1, 2, \ldots, P\}$ **do**
  for $k \in \{1, 2, \ldots, K\}$ **do**
    Calculate the minimum distance of u to each concept space $CS_p^{k,i}$ based on Eq. (15);
    Obtain the similarity matrix S according to Eq. (16);
    Calculate the label of u according to Eq. (17);
  end for
end for
Return the label l_k of u.

where σ is a temperature parameter. The algorithm for assigning a label to the object u is presented in Algorithm 2, and the time complexity is $O(\sum_{p=1}^{P} \sum_{k=1}^{K} |U_k||A_p|)$.

C. Dynamic updating mechanism of concepts

All the concepts learned from a given dataset form a concept lattice, which inherently exhibits multi-granularity characteristic. Intuitively, specialized concepts in the concept lattice reflect the unique features of fewer objects and are more sensitive to data changes, while generalized concepts reflect the common features of more objects and are less sensitive to data changes. Based on this characteristic, by leveraging the extent information of concepts, we can learn concepts at different granularity levels, seeking a balance between common and unique features.

First, the updating mechanism when objects are dynamically increasing is considered. Assume we have already obtained the original multi-view data partition $\{D_p^k | p = 1, 2, \ldots, P, k = 1, 2, \ldots, K\}$, the initial set of clues $\{Clue_p^k | p = 1, 2 \ldots, P, k = 1, 2, \ldots, K\}$, and the optimal granularity level i. Suppose there is an additional set of objects ΔU whose multi-view features are known, but their labels are unknown. We need to update these objects into the multi-view concept space CS.

The proposed updating mechanism can be divided into two parts: 1) Label prediction and clue updating; 2) Concept generation. For 1), the labels of each $u \in \Delta U$ can be predicted according to Algorithm 2. Suppose the label of $u \in \Delta U$ is predicted as l_k, then u is added to the data subsets $\{D_p^k | p = 1, 2, \ldots, P\}$. For any $p \in \{1, 2, \ldots, P\}$, the clues can also be updated as follows:

$$\begin{aligned} &Obj_p^k \leftarrow Obj_p^k \cup \{u\}, \\ &Fea_p^k \leftarrow \bigcup_{a \in A_p} \{\{u' \in U_k \cup \{u\} | C_p^k(u', a) > \frac{\max(C_p^k(:, a))}{2}\}\}. \end{aligned} \tag{20}$$

After giving the predicted labels for all $u \in \Delta U$, for convenience, the added data subset is denoted as ΔD_p^k and the added set of clues is represented as $\Delta Clue_p^k$ for any

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$p \in \{1, 2, \ldots, P\}$ and $k \in \{1, 2, \ldots, K\}$. Then, the updated data subsets and the updated set of clues is expressed as $D_p^k \cup \Delta D_p^k$ and $Clue_p^k \cup \Delta Clue_p^k$.

After updating all newly added objects to the clue set, we proceed to 2), i.e., updating the concepts. We first calculate the initial concept subspace GCS_p^k that learned from the updated data subset $D_p^k \cup \Delta D_p^k$ and the updated set of clues $Clue_p^k \cup \Delta Clue_p^k$. Then, based on Eq. (12), we calculate the similarity between any two concepts within GCS_p^k. Based on the similarity between concepts in GCS_p^k, their update mechanism at granularity level i is provided by the following Property 3. It should be noted that the weight vector W of each view and the granularity level i remain unchanged before and after update.

Property 3. *Let $D_p^k \cup \Delta D_p^k$ and $Clue_p^k \cup \Delta Clue_p^k$ be the updated data subset and the set of clues, and GCS_p^k be the updated initial concept subspace. For any $(E, (I^+, I^-)) \in GCS_p^k$, suppose $(E_1, (I_1^+, I_1^-)), (E_2, (I_2^+, I_2^-)), \ldots, (E_i, (I_i^+, I_i^-))$ are the top-i concepts that are most similar to $(E, (I^+, I^-))$ in GCS_p^k, then:*

$$\mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j) = \Big(\bigcap_{j=1}^{i} \mathcal{L}_p^{k}(E_j), \bigcap_{j=1}^{i} \mathcal{L}_p^{k-}(E_j)\Big), \tag{21}$$

$$\mathcal{F}_p^{k*} \circ \mathcal{L}_p^{k*}(\bigcup_{j=1}^{i} E_j) = \Big\{u \in U_k | \bigcap_{j=1}^{i} \mathcal{L}_p^{k}(E_j) \subseteq \mathcal{L}_p^{k}(\{u\})\Big\} \bigcap \Big\{u \in U_k | \bigcap_{j=1}^{i} \mathcal{L}_p^{k^-}(E_j) \subseteq \mathcal{L}_p^{k^-}(\{u\})\Big\}. \tag{22}$$

Proof. It can be proved from Property 1. □

Eq. (21) and Eq. (22) correspond to the intent and extent of the concept in Eq. (13), respectively. According to Eq. (22), we can learn the intent of the concepts in a more convenient way. Specifically, once the top-i similar concepts to $(E, (I^+, I^-))$ are obtained, the intent of the generated concept can be directly derived by performing the minimum operation in the intents of $\{(E_1, (I_1^+, I_1^-)), (E_2, (I_2^+, I_2^-)), \ldots, (E_i, (I_i^+, I_i^-))\}$. Furthermore, since only the intent is effective for multi-view classification, there is no need to calculate the extents of the concepts for the next step of concept updating, which can further save computational resources. In other words, under the concept-updating mechanism introduced in this subsection, the updated data subset, the updated set of clues, and the optimal granularity level i are the critical elements, while the updated concept subspace can be derived via Property 3. Based on the above discussion, we provide a corresponding algorithm for the generation and updating mechanism of concepts when objects are dynamically added in Algorithm 3, and the intent part of $CS_p^{k,i}$ is denoted as $IntCS_p^{k,i}$.

Second, we discuss the dynamic updating mechanism of concepts when features are dynamically added. Since the concepts are updated based on clues in this paper, only the portion related to clue updates needs to be replaced, while

Algorithm 3 Dynamic updating mechanism of concepts with the increase of objects

Input: The set of clues $\{Clue_p^k | p = 1, 2, \ldots, P, K = 1, 2, \ldots, K\}$, the data partition $\{D_p^k | p = 1, 2, \ldots, P, K = 1, 2, \ldots, K\}$, a set of new objects ΔU, the granularity level i;

Output: The intents of the updated multi-view concept space $IntCS$.

1: Assign the label for each $u \in \Delta U$.
2: **for** $p \in \{1, 2, \ldots, P\}$ **do**
3:   **for** $k \in \{1, 2, \ldots, K\}$ **do**
4:     Obtain the updated data subset $D_p^k \cup \Delta D_p^k$;
5:     Calculate updated set of clues $Clue_p^k \cup \Delta Clue_p^k$;
6:     Learn the initial concept subspace GCS_p^k based on $D_p^k \cup \Delta D_p^k$ and $Clue_p^k \cup \Delta Clue_p^k$;
7:     **for** each $(E, (T^+, I^-)) \in GCS_p^k$ **do**
8:       Find the top-i similar concept with respect to $(E, (T^+, I^-))$;
9:       Calculate the updated intent according to Eq. (21) and add it to $IntCS_p^{k,i}$;
10:     **end for**
11:   **end for**
12: **end for**
13: Return the intents of the updated multi-view concept space $IntCS = \{IntCS_p^{k,i} | p = 1, 2, \ldots, P, k = 1, 2, \ldots, K\}$.

the other parts remain the same as the mechanism for object updates. Let $\{\Delta A_p | p = 1, 2, \ldots, P\}$ be the added features, where ΔA_p is the updated features in the p-th view. For any $p \in \{1, 2, \ldots, P\}$ and $k \in \{1, 2, \ldots, K\}$,

$$Obj_p^k \leftarrow Obj_p^k, \quad Fea_p^k \leftarrow \bigcup_{a \in A_p \cup \Delta A_p} \{\{u \in U_k | C_p^k(u, a) > \frac{\max(C_p^k(:, a))}{2}\}\}. \tag{23}$$

Then, after adding the new features to the corresponding positions in each data subset D_p^k, the other parts are updated in the same way as objects, as in lines 2–13 of Algorithm 3.

The advantages of the dynamic updating mechanism for multi-view concepts presented in this subsection are reflected in the following aspects: 1) Greater flexibility: Compared to other CCL models, it updates based on dynamic clues, making it applicable to any concept update scenario, such as the addition of objects, the addition of features, or both simultaneously. 2) Higher efficiency: Compared to more complex concept updating mechanisms, it makes full use of the optimal parameter i, allowing for quick computation of the intents of the updated concepts based on Property 3.

The overall process of DMVCCL can be divided into the following three parts: 1) Constructing the multi-view concept space based on Subsection III-A; 2) Dynamically updating concepts in response to the dynamically increasing objects and features, based on the discussion in this subsection; 3) Predicting labels for new objects based on the discussion in

TABLE I
DETAILED INFORMATION OF THE MULTI-VIEW DATASETS

ID	Dataset	Objects	Views	Labels
MF	Multiple features[1]	2000	$(216, 76, 64, 6, 240, 47)$	10
100L	100leaves[2]	1600	$(64, 64, 64)$	100
LU	Land use 21[2]	2100	$(20, 59, 40)$	21
Pie	Pieface[2]	680	$(484, 256, 279)$	68
TM	Two moon[2]	200	$(2, 2)$	2
ALOI	Aloi100[2]	10800	$(77, 13, 64, 12)$	100
TR	Tree ring[2]	300	$(2, 2)$	3
COIL	Coil100[2]	7200	$(30, 99, 30)$	100
Minist	Minist[2]	10000	$(30, 9, 30)$	10
Wiki	Wikipedia [41]	2866	$(1024, 100)$	10

Subsection III-B. To facilitate understanding, a diagram is presented in Fig. 1 to illustrate the proposed DMVCCL model.

IV. Experiments

A series of experiments is conducted to demonstrate that the proposed DMVCCL model can effectively represent and fuse knowledge from multi-view data.

A. Experimental setup

Ten widely used multi-view classification datasets are used in this paper, with detailed information presented in Table I. In Table I, "Objects" denotes the number of objects in multi-view datasets, "Views" indicates both the count of views and the number of features in each view, and "Labels" represents the number of labels. For example, for LU dataset, it contains 2100 objects, 21 labels, and three views, each containing 20, 59, and 40 features, respectively. All feature values in these datasets are continuous. To obtain fuzzy data, they are normalized to the interval $[0.1, 0.9]$ using the following equation:

$$C(u,a) = 0.1 + (0.9-0.1)\frac{C'(u,a)-\min(C'(:,a))}{\max(C'(:,a))-\min(C'(:,a))}, \tag{24}$$

where $\max(C'(:,a))$ and $\min(C'(:,a))$ are the maximum and minimum values of feature a in the original dataset, respectively.

Four typical metrics for measuring classification performance are introduced: accuracy(ACC), precision(PRE), recall(REC), and F1-score($F1$), where

$$\begin{aligned} ACC &= \frac{TP+TN}{FP+FN+TP+TN}, \\ PRE &= \frac{TP}{TP+FP}, \\ REC &= \frac{TP}{TP+FN}, \\ F1 &= \frac{2\times REC\times PRE}{REC+PRE}. \end{aligned} \tag{25}$$

TP is the number of true positive objects, TN is the number of true negatives objects, FP is the number of false positive objects and FN is the number of false negative objects. The larger the value of the above metric, the better the classification performance.

The experiments are conducted on a personal computer with Processor: Intel(R) Core(TM) i7-13700H; Memory: 32 GB; Programming language: MATLAB R2020b.

B. Comparisons of static multi-view classification performance

To demonstrate the effectiveness of the proposed method, comparisons are made with the following two groups of methods:

1) Four multi-view learning methods, including: (1) Multi-view fuzzy concept-cognitive learning (MVFCCL) [28], it establishes a framework for representing and integrating knowledge in multi-view data using concepts as carriers. (2) Association-based fusion method for multi-modal classification (AF) [3], it provides an association-based fusion method that simultaneously models the high-order and correlation information of multi-view data. (3) Multi-view classification with cohesion and diversity (MCCD) [42], it improves classification performance by mining the consistent and complementary information in multi-view data. (4) Independent prototypes formed multi-view sub-space clustering (IPMVSC) [6], it constructs an information-granule-based multi-view TSK fuzzy classification model.
2) Three CCL models, including: (1) Fuzzy granular three-way concept-cognitive learning (F3WG-CCL) [43], it learns fuzzy three-way granular concepts form data and utilize big concepts as knowledge carriers. (2) Incremental learning mechanism based on progressive fuzzy three-way concept (ILMPFTC) [25], it constructs clues based on the distances between objects, and learns progressive concepts by fusing similar concepts to capture the knowledge in the data. (3) Memory-based concept-cognitive learning (MFCCL) [26], it constructs the initial learning clues using cosine similarity and designs mechanisms for concept forgetting and recall based on the extent information.

For AF, after obtaining the fused features, we concatenate the data from all views and use the resulting combined features as input to a classifier. We employ decision tree (DT) and support vector machine (SVM) as base classifiers for AF, referred to as AF-DT-C and AF-SVM-C, respectively. Additionally, we report the results obtained by directly concatenating all features and supplying them to DT and SVM classifiers, denoted as DT-C and SVM-C, respectively. For F3WG-CCL, MFCCL, and ILMPFTC, we adopt the same feature-concatenation strategy, and these CCL models are then performed on the concatenated data to obtain the classification results. The proposed method in this paper requires two parameters, σ and i. σ controls the weight of each view and it is selected in $\{10^{-3}, 10^{-2}, 10^{-1}, 0, 1, 10^{1}, 10^{2}, 10^{3}\}$. When $\sigma = 0$, specify $W(p) = 1$ for all views in Eq. (19). i represents the granularity level, which is adjusted in $\{1, 2, 3, 4, 5\}$. To ensure fairness, all experiments are conducted using the same

[1] http://archive.ics.uci.edu/dataset/72/multiple+features
[2] https://github.com/JethroJames/Awesome-Multi-View-Learning-Datasets?tab=readme-ov-file

TABLE II
ACCURACY COMPARISON WITH MULTI-VIEW LEARNING MODELS

Dataset	DMVCCL	MCCD	MVFCCL	AF-SVM-C	AF-DT-C	IPMVSC
MF	**98.95±0.37**	98.00±0.75	98.30±0.79	97.05±1.26	95.30±1.84	98.85±0.58
100L	**99.44±0.55**	84.56±2.21	98.06±0.95	90.69±2.35	76.00±2.64	6.75±1.55
LU	**75.29±3.11**	44.76±4.44	73.95±3.20	65.62±3.89	46.57±3.60	63.24±3.66
Pie	87.35±4.11	**93.38±3.75**	87.79±3.92	57.06±7.36	32.50±5.39	81.03±4.83
TM	**100.00±0.00**	91.00±6.15	74.50±14.23	88.00±7.53	98.50±4.74	97.00±4.22
ALOI	**99.63±0.18**	80.87±1.49	98.95±0.31	96.90±0.58	92.80±0.89	74.65±4.59
TR	**100.00±0.00**	60.00±10.89	54.00±9.66	60.00±10.89	96.67±5.67	70.00±9.43
COIL	99.75±0.22	80.87±1.49	99.81±0.15	99.22±0.32	84.94±1.39	**99.82±0.15**
Minist	**88.32±1.11**	79.35±1.16	86.88±1.28	86.79±1.33	76.60±1.75	87.29±1.41
Wiki	80.78±2.05	**82.62±1.47**	81.23±2.05	74.63±2.58	73.52±1.73	75.64±2.31

data partitioning with ten-fold cross-validation, and the mean values and standard deviations are recorded.

The classification accuracy, precision, recall, and F1-score of the DMVCCL and five multi-view classification methods are shown in Tables II-V. Within each table, the best classification result for each dataset is represented in bold. As summarized across all metrics, DMVCCL achieves the best classification performance on eight of ten datasets, followed by MCCD on two datasets and IPMVSC on one dataset. The experimental results confirm the effectiveness of DMVCCL for multi-view classification.

The classification performance of DMVCCL compared with single-view CCL models is presented in Tables VI-IX, and they represent classification accuracy, precision, recall, and F1-score, respectively. It can be observed that DMVCCL achieves the best classification performance on 10 datasets, and the three CCL models all achieve the best classification performance on TM and TR datasets. The experimental results demonstrate that DMVCCL outperforms single-view CCL models with concatenation fusion method.

To ascertain the statistical significance of the differences among these methods, the Friedman test [44] is employed. Based on the experimental results in Tables II to IX, the calculated p-values for accuracy, precision, recall and F1-score are 3.9231×10^{-5}, 2.9076×10^{-5}, 1.9126×10^{-5}, and 2.1105×10^{-5}, respectively. These p-value are all below 0.05, indicating a significant difference in classification performance among these methods at the 0.05 significance level. Subsequently, the Nemenyi post hoc test [45] is utilized to elucidate the differences between each pair of methods. The corresponding critical difference diagrams are presented in Fig. 3, where the horizontal axis denotes the average rank of each method. Methods that are not connected by horizontal lines are deemed to exhibit significant differences. The aforementioned analysis demonstrates that DMVCCL achieves remarkable performance for multi-view classification.

C. Performance evaluation with objects dynamic increase

In this subsection, we aim to demonstrate that the proposed clue-based dynamic classification model outperforms existing CCL methods. To our knowledge, few existing CCL methods can handle dynamic multi-view data. Hence, we compare our proposed method with single-view CCL methods, including F3WGCCL, MFCCL, and ILMPFTC. Since these CCL models are limited to single-view data, we still employ feature concatenation to transform multi-view data into a dataset that can be processed by these single-view CCL methods.

The experimental setup is as follows: Firstly, to simulate a dynamic environment, the objects in the multi-view dataset are first partitioned into ten equal parts. Five of these parts are used as the initial training data to learn an initial multi-view concept space, while the remaining five are treated as incremental data blocks that arrive sequentially over different time periods. Notably, the labels of these five sequentially added blocks are unknown. Then, the remaining five data blocks are then treated as sequentially arriving data at times t_1 to t_5. For each data block, we assign labels using the multi-view concept space learned in the preceding stage together with the proposed multi-view classification method. Based on the updated data, the multi-view concept space is then updated. In this process, the classification performance for each data block is computed by comparing the ground-truth labels and the predicted labels. The classification accuracy, precision, recall, and F1-score at each time point are recorded.

The experimental results are reported in Tables X, XI, XII and XIII, which present classification accuracy, precision, recall, and F1-score, respectively. The notation "Ave.±Std." represents the average value and standard deviation calculated over the 5 time points for each metric, and the best average classification performance is highlighted in bold. The comparison of classification accuracy at each time point is illustrated in Fig. 4, which highlights the performance differences across time points. According to the experimental results, DMVCCL achieves the best classification performance on 8 out of the 10 datasets, MFCCL achieves the best classification performance on TM and TR datasets, and ILMPFTC achieves the best classification performance on TR datasets. The results demonstrate the effectiveness of DMVCCL in dynamic environments.

It should be noticed that for the three CCL models, we employ feature concatenation as the fusion strategy. The advantage is that it fully exploiting the information from each view and achieve better classification performance. Moreover, this fusion strategy delivers better classification results when the number of views or features is small, such as TM and TR datasets. When either the number of features or views are large, their performance is weaker. One of the reasons is that CCL models exhibit poor classification performance for high-dimensional data. We also noticed that MFCCL and ILMPFTC have similar classification performance. This is because when there are many features, the clues generated by

TABLE III
PRECISION COMPARISON WITH MULTI-VIEW LEARNING MODELS

Dataset	DMVCCL	MCCD	MVFCCL	AF-SVM-C	AF-DT-C	IPMVSC
MF	**98.96±0.39**	97.91±0.80	98.26±0.93	97.10±1.26	95.33±1.86	98.83±0.47
100L	**99.07±1.38**	82.45±3.26	97.18±1.54	87.91±3.23	72.26±4.91	6.68±1.38
LU	**76.28±2.12**	46.79±5.17	74.95±3.31	66.74±2.94	47.94±2.55	64.24±3.21
Pie	84.09±6.38	**92.32±3.80**	84.96±6.34	50.53±7.45	27.24±5.47	78.76±5.22
TM	**100.00±0.00**	91.68±5.53	77.89±12.82	88.16±7.84	98.45±4.89	97.52±5.90
ALOI	**99.69±0.15**	83.32±1.66	98.95±0.30	96.99±0.60	93.14±0.82	77.88±3.50
TR	**100.00±0.00**	20.00±3.63	59.66±7.64	20.00±3.63	97.02±5.68	54.50±4.96
COIL	99.77±0.22	83.32±1.66	99.83±0.13	99.26±0.27	86.12±1.51	**99.85±0.11**
Minist	**88.34±1.09**	80.23±1.05	86.88±1.23	86.83±1.23	76.64±1.80	87.40±1.33
Wiki	80.76±2.91	84.06±1.66	80.98±1.98	75.09±2.21	73.64±1.79	**85.70±2.20**

TABLE IV
RECALL COMPARISON WITH MULTI-VIEW LEARNING MODELS

Dataset	DMVCCL	MCCD	MVFCCL	AF-SVM-C	AF-DT-C	IPMVSC
MF	**98.98±0.31**	98.05±0.83	98.28±0.80	97.09±1.37	95.51±1.84	98.91±0.55
100L	**99.07±1.19**	82.13±4.10	97.32±1.98	87.34±3.25	72.10±5.71	9.27±1.83
LU	**75.47±2.20**	45.48±3.07	74.17±2.74	65.50±3.19	46.92±3.17	64.06±2.73
Pie	83.79±4.96	**92.17±4.03**	84.85±5.10	51.23±8.53	28.92±5.52	75.66±5.50
TM	**100.00±0.00**	90.76±6.43	73.92±13.84	87.72±8.29	98.19±5.73	97.18±3.88
ALOI	**99.65±0.19**	81.23±1.19	99.00±0.29	96.86±0.66	92.88±0.72	75.07±5.04
TR	**100.00±0.00**	33.33±0.00	67.00±7.91	33.33±0.00	95.81±8.66	66.67±0.00
COIL	99.75±0.23	81.23±1.19	99.80±0.19	99.20±0.33	85.28±1.75	**99.84±0.12**
Minist	**88.35±1.01**	79.39±0.97	86.90±1.07	86.82±1.23	76.64±1.70	87.32±1.26
Wiki	79.86±1.97	**80.84±1.17**	80.65±1.71	72.75±3.05	72.64±1.63	73.93±2.37

TABLE V
F1-SCORE COMPARISON WITH MULTI-VIEW LEARNING MODELS

Dataset	DMVCCL	MCCD	MVFCCL	AF-SVM-C	AF-DT-C	IPMVSC
MF	**98.94±0.35**	97.93±0.83	98.24±0.88	97.01±1.34	95.29±1.94	98.83±0.58
100L	**98.99±1.31**	79.64±4.17	97.01±1.83	86.36±3.33	69.63±5.23	6.52±1.15
LU	**74.39±2.15**	38.84±3.78	72.90±3.03	64.86±2.93	45.96±2.81	61.60±3.37
Pie	82.54±5.78	**91.41±4.22**	83.51±5.81	48.69±8.03	26.10±5.00	75.16±5.56
TM	**100.00±0.00**	90.46±6.51	72.04±15.57	87.40±8.10	98.29±5.41	96.94±4.32
ALOI	**99.65±0.18**	78.95±1.58	98.92±0.30	96.77±0.67	92.57±0.82	72.94±5.24
TR	**100.00±0.00**	24.83±2.75	50.27±10.08	24.83±2.75	96.07±8.02	58.85±4.11
COIL	99.73±0.26	78.95±1.58	99.78±0.20	99.14±0.33	84.70±1.69	**99.83±0.13**
Minist	**88.27±1.06**	78.98±1.06	86.69±1.19	86.68±1.24	76.54±1.73	87.15±1.32
Wiki	79.81±2.28	**81.58±1.16**	80.33±2.07	73.24±2.60	72.60±1.39	75.64±2.31

TABLE VI
ACCURACY COMPARISON WITH CONCEPT-COGNITIVE LEARNING MODELS AND CLASSICAL CLASSIFICATION METHODS

Dataset	DMVCCL	MFCCL-C	F3WGCCL-C	ILMPFTC-C	DT-C	SVM-C
MF	**98.95±0.37**	98.05±0.55	97.25±0.72	98.05±0.55	93.50±2.04	98.15±0.75
100l	**99.44±0.55**	98.69±0.69	96.06±1.62	98.81±0.69	68.06±4.47	96.37±1.64
LU	**75.29±3.11**	69.71±2.90	25.00±2.50	69.71±2.90	46.14±3.75	55.19±3.59
Pie	**87.35 ±4.11**	79.12±7.49	58.09±7.38	79.12±7.49	55.29±6.05	86.32±6.32
TM	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	96.50±3.37	88.00±7.53
ALOI	**99.63±0.18**	98.56±0.41	91.46±0.63	98.56±0.41	84.83±0.71	96.31± 0.86
TR	**100.00±0.00**	**100.00±0.00**	73.67±10.24	**100.00±0.00**	98.33±3.24	60.00±10.89
COIL	**99.75±0.22**	98.68±0.45	83.92±1.18	98.82±0.38	81.53±1.95	99.15±0.32
Minist	**88.32±1.11**	86.14±1.11	77.04±2.31	86.14±1.11	76.74±1.27	86.41±1.24
Wiki	**80.78±2.05**	32.13±1.98	27.07±3.37	32.13±1.98	65.21±1.75	76.69±2.35

TABLE VII
PRECISION COMPARISON WITH CONCEPT-COGNITIVE LEARNING MODELS AND CLASSICAL CLASSIFICATION METHODS

Dataset	DMVCCL	MFCCL-C	F3WGCCL-C	ILMPFTC-C	DT-C	SVM-C
MF	**98.96±0.39**	98.13±0.55	97.32±0.79	98.13±0.55	93.61±2.14	98.16±0.86
100l	**99.07±1.38**	97.95±1.52	94.08±2.06	98.47±1.20	65.13±5.89	96.03±2.25
LU	**76.28±2.12**	71.65±2.53	38.83±4.97	71.67±3.02	47.23±3.39	60.05±2.57
Pie	**84.09±6.38**	74.41±7.93	55.92±10.72	74.41±7.93	48.72±5.59	83.70±7.68
TM	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	96.64±3.42	88.16±7.84
ALOI	**99.69±0.15**	98.58±0.38	93.48±0.83	98.59±0.35	84.83±0.74	96.50±0.82
TR	**100.00±0.00**	**100.00±0.00**	77.51±17.03	**100.00±0.00**	98.39±3.22	20.00±3.63
COIL	**99.77±0.22**	98.83±0.38	92.06±1.23	98.95±0.31	82.59±1.69	99.19±0.32
Minist	**88.34±1.09**	86.19±1.16	81.13±1.64	86.19±1.16	76.84±1.32	86.43±1.15
Wiki	**80.76±2.91**	30.62±2.08	34.74±5.70	30.62±2.08	64.96±1.56	77.93±2.15

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TABLE VIII
RECALL COMPARISON WITH CONCEPT-COGNITIVE LEARNING MODELS AND CLASSICAL CLASSIFICATION METHODS

Dataset	DMVCCL	MFCCL-C	F3WGCCL-C	ILMPFTC-C	DT-C	SVM-C
MF	**98.98±0.31**	98.15±0.60	97.40±0.69	98.15±0.60	93.84±2.28	98.17±0.81
100L	**99.07±1.19**	97.75±1.52	94.57±2.31	98.32±1.01	63.16±5.24	95.79±2.11
LU	**75.47±2.20**	69.80±1.75	24.95±2.46	69.80±1.75	46.73±3.97	55.43±2.97
Pie	**83.79±4.96**	74.25±7.64	54.59±8.52	74.25±7.64	50.53±4.70	83.64±7.18
TM	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	96.26±3.67	87.72±8.29
ALOI	**99.65±0.19**	98.56±0.34	91.35±0.72	98.56±0.33	84.83±0.81	96.28±0.88
TR	**100.00±0.00**	**100.00±0.00**	71.56±9.27	**100.00±0.00**	97.90±5.20	33.33±0.00
COIL	**99.75±0.23**	98.71±0.42	83.97±1.10	98.82±0.38	81.72±1.66	99.17±0.26
Minist	**88.35±1.01**	86.18±1.07	77.09±1.81	86.18±1.07	76.74±1.16	86.43±1.11
Wiki	**79.86±1.97**	30.06±2.33	24.79±2.84	30.06±2.33	64.73±1.65	74.77±2.46

TABLE IX
F1-SCORE COMPARISON WITH CONCEPT-COGNITIVE LEARNING MODELS AND CLASSICAL CLASSIFICATION METHODS

Dataset	DMVCCL	MFCCL-C	F3WGCCL-C	ILMPFTC-C	DT-C	SVM-C
MF	**98.94±0.35**	98.09±0.58	97.26±0.75	98.09±0.58	93.47±2.30	98.12±0.84
100L	**98.99±1.31**	97.64±1.53	93.78±2.34	98.15±1.08	61.24±5.50	95.26±2.32
LU	**74.39±2.15**	68.43±1.89	21.51±2.56	68.43±1.89	45.69±3.50	55.55±2.80
Pie	**82.54±5.78**	72.47±8.23	52.37±9.13	72.47±8.23	47.15±5.25	82.18±7.72
TM	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	**100.00±0.00**	96.30±3.50	87.40±8.10
ALOI	**99.65±0.18**	98.49±0.36	90.79±0.57	98.50±0.34	84.83±0.87	96.15±0.91
TR	**100.00±0.00**	**100.00±0.00**	69.42±9.49	**100.00±0.00**	98.07±4.35	24.83±2.75
COIL	**99.73±0.26**	98.61±0.45	84.69±1.02	98.73±0.39	81.04±1.74	99.09±0.34
Minist	**88.27±1.06**	86.10±1.13	76.75±2.09	86.10±1.13	76.70±1.24	86.29±1.13
Wiki	**79.81±2.28**	29.73±2.29	22.11±2.88	29.73±2.29	64.33±1.54	75.49±2.34

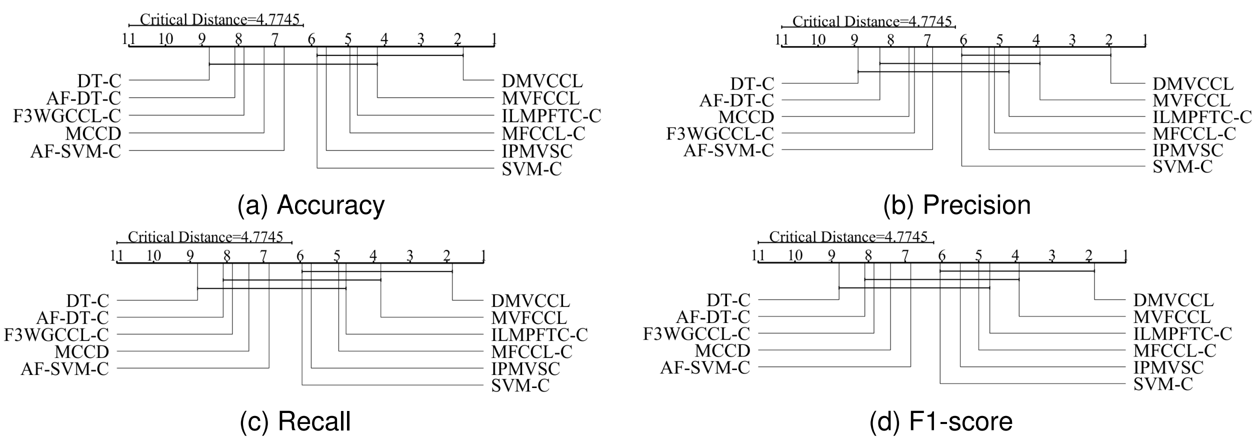

Fig. 3. Nemenyi post-hoc test of DMVCCL against other comparing methods with respect to classification performance at the 0.05 significance level

cosine similarity and those generated by Euclidean distance tend to be similar. Whereas DMVCCL first learn concepts under each view, and then integrates them by consider the representational capacity of the concepts under each view. In this way, the knowledge in multi-view data can be represented and integrated accurately with concepts as carriers.

V. CONCLUSION

This paper has proposed a concept-cognitive learning model for dynamic multi-view data. Firstly, based on the definition of multi-view concept space, a knowledge representation method for multi-view data has been established. It uses fuzzy three-way concepts as the basic carriers of knowledge, and simultaneously uses positive information and complementary information to depict the relationship between objects and features. The similarity between concepts have been defined based on the extent distance and the intent distance between them. Based on the similarity of concepts, by continuously integrating similar concepts, the concepts at different granularity levels are obtained. This process simulates the characteristic of human cognition, which progresses from the concrete to the abstract. Besides, the representation capability of concepts learned under different views has been used to determine the weight of each view. On this basis, the weight vector of each view and the similarity between concepts and data within each view have been considered for multi-view classification. Furthermore, a clue-based dynamic update mechanism for concepts have been developed to handle incremental multi-view data by accounting for the varying sensitivity of concepts at different granularity levels. Finally, some experiments have been conducted to validate the effectiveness of the proposed DMVCCL model.

It is noted that the definition of the extent of concepts in this paper is classical, meaning that an object either belongs to

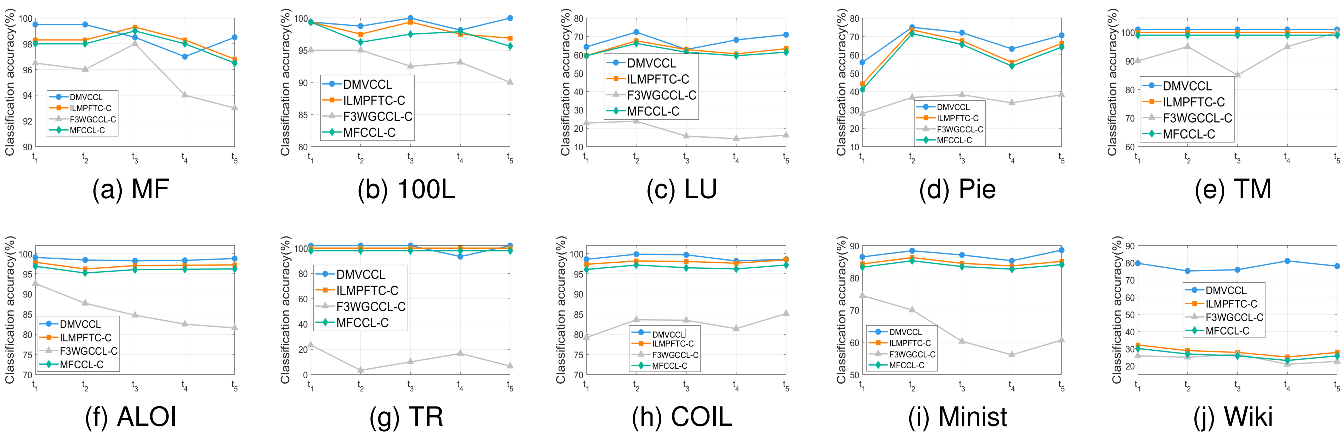

Fig. 4. Classification accuracy comparison with the dynamic increase of objects on 10 datasets

TABLE X
COMPARISON OF ACCURACY AS THE NUMBER OF OBJECTS INCREASES

Dataset	Metrics	t_1	t_2	t_3	t_4	t_5	Ave.±Std.
MF	DMVCCL	98.50	98.50	98.50	97.00	97.50	**98.00±0.71**
	ILMPFTC	98.00	98.00	99.00	98.00	96.50	97.90±0.89
	F3WGCCL	96.50	96.00	98.00	94.00	93.00	95.50±2.00
	MFCCL	98.00	98.00	99.00	98.00	96.50	97.90±0.89
100L	Our	99.38	98.75	100.00	98.13	100.00	**99.25±0.81**
	ILMPFTC	99.38	97.50	99.38	97.50	96.88	98.13±1.17
	F3WGCCL	95.00	95.00	92.50	93.13	90.00	93.13±2.07
	MFCCL	99.38	96.25	97.50	97.88	95.63	97.13±1.44
LU	Our	64.29	72.38	62.86	68.10	70.95	**67.71±4.12**
	ILMPFTC	59.52	67.62	62.86	60.48	63.33	62.76±3.15
	F3WGCCL	22.86	23.81	15.71	14.29	16.19	18.57±4.42
	MFCCL	59.52	66.19	61.43	60.48	61.43	61.81±2.57
Pie	Our	55.88	75.00	72.06	63.24	70.59	**67.35±7.74**
	ILMPFTC	44.12	73.53	67.65	55.88	66.18	61.47±11.60
	F3WGCCL	27.94	36.76	38.24	33.82	38.24	35.00±4.34
	MFCCL	44.12	73.53	67.65	55.88	66.18	61.47±11.60
TM	Our	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	90.00	95.00	85.00	95.00	100.00	93.00±5.70
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
ALOI	Our	99.07	98.43	98.24	98.33	98.80	**98.57±0.35**
	ILMPFTC	97.87	96.20	97.04	97.13	97.22	97.09±0.60
	F3WGCCL	92.59	87.69	84.72	82.50	81.57	85.81±4.46
	MFCCL	97.87	96.20	97.04	97.13	97.22	97.09±0.60
TR	Our	100.00	100.00	100.00	93.33	100.00	98.67±2.98
	ILMPFTC	100.00	100.00	100.00	93.33	100.00	98.67±2.98
	F3WGCCL	23.33	3.33	10.00	16.67	6.67	12.00±8.03
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
COIL	Our	98.61	99.86	99.72	98.19	98.61	**99.00±0.74**
	ILMPFTC	97.36	98.19	98.06	97.64	98.47	97.94±0.44
	F3WGCCL	79.17	83.61	83.47	81.39	85.14	82.56±2.32
	MFCCL	97.08	98.19	97.50	97.22	98.19	97.64±0.53
Minist	Our	86.50	88.40	87.10	85.30	88.60	**87.18±1.37**
	ILMPFTC	84.30	86.30	84.50	83.70	85.10	84.78±0.99
	F3WGCCL	74.50	70.00	60.30	56.10	60.70	64.32±7.63
	MFCCL	84.30	86.30	84.50	83.70	85.10	84.78±0.99
Wiki	Our	79.72	75.26	75.96	81.12	78.05	**78.02±2.47**
	ILMPFTC	32.17	28.92	27.87	25.17	27.87	28.40±2.52
	F3WGCCL	25.87	25.09	26.83	20.98	22.65	24.28±2.41
	MFCCL	32.17	28.92	27.87	25.17	27.87	28.40±2.52

TABLE XI
COMPARISON OF PRECISION AS THE NUMBER OF OBJECTS INCREASES

Dataset	Metrics	t_1	t_2	t_3	t_4	t_5	Ave.±Std.
MF	DMVCCL	97.99	98.61	97.76	97.46	97.82	**97.93±0.43**
	ILMPFTC	97.49	98.14	98.64	97.93	96.54	97.74±0.79
	F3WGCCL	96.39	96.60	98.05	94.64	92.74	95.68±2.04
	MFCCL	97.49	98.14	98.64	97.93	96.54	97.74±0.79
100L	DMVCCL	99.58	97.08	100.00	96.30	100.00	**98.59±1.77**
	ILMPFTC	99.58	95.41	98.42	97.36	95.88	97.33±1.73
	F3WGCCL	93.13	89.58	90.72	89.51	87.69	90.12±2.00
	MFCCL	99.37	92.84	97.22	96.50	96.41	96.47±2.35
LU	DMVCCL	68.85	74.32	66.54	71.72	73.34	**70.95±3.22**
	ILMPFTC	64.64	67.01	68.32	67.21	66.43	66.72±1.35
	F3WGCCL	43.45	35.94	28.32	26.17	20.99	30.97±8.81
	MFCCL	64.60	65.32	67.79	66.88	65.17	65.95±1.33
Pie	DMVCCL	48.20	65.03	67.63	61.79	64.47	**61.42±7.68**
	ILMPFTC	40.10	63.83	65.00	53.94	61.95	56.96±10.37
	F3WGCCL	26.24	33.50	37.86	37.79	32.54	33.59±4.77
	MFCCL	40.10	63.83	65.00	53.94	61.95	56.96±10.37
TM	DMVCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	90.00	93.75	85.42	95.45	100.00	92.92±5.52
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
ALOI	DMVCCL	99.19	98.57	98.37	98.55	98.88	**98.71±0.32**
	ILMPFTC	98.00	96.32	97.14	97.09	97.14	97.14±0.60
	F3WGCCL	93.91	90.07	89.94	88.03	86.14	89.62±2.89
	MFCCL	98.00	96.32	97.14	97.09	97.14	97.14±0.60
TR	DMVCCL	100.00	100.00	100.00	95.56	100.00	99.11±1.99
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	19.94	5.56	5.26	5.56	2.67	7.80±6.89
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
COIL	DMVCCL	98.90	99.83	99.69	98.33	98.71	**99.09±0.65**
	ILMPFTC	97.84	97.81	98.23	97.82	98.64	98.07±0.36
	F3WGCCL	89.27	88.06	88.12	86.04	88.87	88.07±1.25
	MFCCL	97.53	97.93	97.73	97.49	98.31	97.80±0.34
Minist	DMVCCL	86.35	88.32	87.17	85.44	88.36	**87.13±1.27**
	ILMPFTC	84.06	86.45	84.51	83.96	84.98	84.79±1.01
	F3WGCCL	78.16	78.80	76.01	76.70	77.20	77.37±1.12
	MFCCL	84.06	86.45	84.51	83.96	84.98	84.79±1.01
Wiki	DMVCCL	80.61	74.26	75.43	80.43	79.87	**78.12±3.03**
	ILMPFTC	31.02	28.19	26.68	25.72	25.09	27.34±2.37
	F3WGCCL	32.30	39.34	34.08	24.90	24.91	31.10±6.22
	MFCCL	31.02	28.19	26.68	25.72	25.09	27.34±2.37

the extent of a concept or does not belong to it. This limitation results in a relatively poor representation ability of the learned concepts, especially when dealing with high-dimensional data. Therefore, exploring the CCL model when the degree to which an object belongs to a concept is represented as a fuzzy value is a worthwhile research issue. Additionally, some multi-view data may have missing values in practical scenarios, which can hinder concept learning. This paper takes advantage of the inherent multi-granularity characteristic of concepts and proposes a strategy for learning concepts at different granularity levels. Intuitively, concepts at different granularity levels are sensitive to missing data to varying degrees. For

TABLE XII
COMPARISON OF RECALL AS THE NUMBER OF OBJECTS INCREASES

Dataset	Metrics	t_1	t_2	t_3	t_4	t_5	Ave.±Std.
MF	DMVCCL	98.29	98.42	98.64	96.57	98.01	97.99±0.82
	ILMPFTC	98.08	97.90	99.05	97.92	97.13	**98.02±0.69**
	F3WGCCL	96.82	96.16	96.87	95.01	92.34	95.44±1.89
	MFCCL	98.08	97.90	99.05	97.92	97.13	**98.02±0.69**
100L	DMVCCL	99.58	97.08	100.00	97.22	100.00	**98.78±1.49**
	ILMPFTC	99.37	94.55	98.73	96.75	96.27	97.14±1.94
	F3WGCCL	93.88	89.06	88.55	91.52	88.35	90.27±2.38
	MFCCL	99.75	92.95	95.47	97.49	95.64	96.26±2.53
LU	DMVCCL	67.51	69.87	64.33	70.41	70.72	**68.57±2.68**
	ILMPFTC	61.38	63.46	62.41	63.14	64.47	62.97±1.16
	F3WGCCL	24.16	20.14	15.21	16.44	15.79	18.35±3.78
	MFCCL	61.63	62.61	62.28	63.14	59.72	61.88±1.33
Pie	DMVCCL	51.63	68.95	63.14	59.72	66.67	**62.02±6.78**
	ILMPFTC	40.99	68.33	63.00	51.67	62.58	57.31±10.95
	F3WGCCL	27.48	32.03	33.33	36.46	36.06	33.07±3.63
	MFCCL	40.99	68.33	63.00	51.67	62.58	57.31±10.95
TM	DMVCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	90.00	96.15	84.34	95.00	100.00	93.10±6.06
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
ALOI	DMVCCL	98.95	98.45	98.47	98.33	98.69	**98.57±0.25**
	ILMPFTC	97.64	96.58	97.21	96.92	97.17	97.10±0.39
	F3WGCCL	92.53	87.27	84.67	84.43	81.34	86.05±4.19
	MFCCL	97.64	96.58	97.21	96.92	97.17	97.10±0.39
TR	DMVCCL	100.00	100.00	100.00	86.67	100.00	97.33±5.96
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	38.89	33.33	33.33	33.33	33.33	34.44±2.48
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
COIL	DMVCCL	98.75	99.80	99.63	98.29	98.35	**98.96±0.71**
	ILMPFTC	97.50	98.17	97.78	97.68	98.38	97.90±0.36
	F3WGCCL	78.91	82.67	83.99	82.72	83.44	82.35±2.00
	MFCCL	97.34	98.06	97.30	97.34	98.08	97.62±0.41
Minist	DMVCCL	86.38	88.29	87.10 8	5.71	88.40	**87.17±1.17**
	ILMPFTC	84.07	86.21	84.54	84.03	84.93	84.76±0.89
	F3WGCCL	74.45	69.38	60.02	57.65	60.18	64.34±7.22
	MFCCL	84.07	86.21	84.54	84.03	84.93	84.76±0.89
Wiki	DMVCCL	78.68	73.80	75.08	78.48	75.99	**76.40±2.13**
	ILMPFTC	31.43	27.26	26.09	23.99	23.78	26.51±3.11
	F3WGCCL	26.78	22.60	22.55	18.80	19.01	21.95±3.27
	MFCCL	31.43	27.26	26.09	23.99	23.78	26.51±3.11

TABLE XIII
COMPARISON OF F1-SCORE AS THE NUMBER OF OBJECTS INCREASES

Dataset	Metrics	t_1	t_2	t_3	t_4	t_5	Ave.±Std.
MF	DMVCCL	98.13	98.49	98.11	96.87	97.86	**97.89±0.61**
	ILMPFTC	97.69	97.97	98.81	97.87	96.71	97.81±0.75
	F3WGCCL	96.36	96.31	97.26	94.13	92.19	95.25±2.06
	MFCCL	97.69	97.97	98.81	97.87	96.71	97.81±0.75
100L	DMVCCL	99.49	97.03	100.00	96.58	100.00	**98.62±1.68**
	ILMPFTC	99.32	94.68	98.55	96.34	95.69	96.92±1.96
	F3WGCCL	92.65	88.86	88.50	89.53	86.22	89.15±2.32
	MFCCL	99.44	92.36	96.04	96.49	94.99	95.86±2.56
LU	DMVCCL	65.35	69.66	63.39	69.40	70.19	**67.60±3.04**
	ILMPFTC	59.63	63.36	62.31	62.69	64.08	62.41±1.69
	F3WGCCL	21.64	18.10	13.38	13.45	12.55	15.82±3.92
	MFCCL	59.59	61.99	61.89	62.27	59.84	61.12±1.29
Pie	DMVCCL	47.12	65.29	62.44	58.02	63.84	**59.34±7.35**
	ILMPFTC	37.49	64.40	60.87	50.30	60.20	54.65±10.93
	F3WGCCL	25.79	30.97	32.89	33.76	31.98	31.08±3.14
	MFCCL	37.49	64.40	60.87	50.30	60.20	54.65±10.93
TM	DMVCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	90.00	94.67	84.65	94.99	100.00	92.86±5.79
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
ALOI	DMVCCL	98.99	98.43	98.33	98.35	98.72	**98.56±0.28**
	ILMPFTC	97.67	96.30	97.04	96.84	96.97	96.96±0.49
	F3WGCCL	91.98	86.08	83.05	82.15	79.67	84.58±4.73
	MFCCL	97.67	96.30	97.04	96.84	96.97	96.96±0.49
TR	DMVCCL	100.00	100.00	100.00	89.29	100.00	97.86±4.79
	ILMPFTC	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
	F3WGCCL	26.20	9.52	9.09	9.52	4.94	11.85±8.25
	MFCCL	100.00	100.00	100.00	100.00	100.00	**100.00±0.00**
COIL	DMVCCL	98.68	99.80	99.63	98.19	98.35	**98.93±0.74**
	ILMPFTC	97.44	97.83	97.72	97.63	98.32	97.79±0.33
	F3WGCCL	80.43	82.28	82.64	81.34	83.05	81.95±1.06
	MFCCL	97.20	97.83	97.19	97.23	97.99	97.49±0.39
Minist	DMVCCL	86.26	88.23	87.09	85.41	88.34	**87.07±1.26**
	ILMPFTC	83.99	86.18	84.48	83.85	84.81	84.66±0.93
	F3WGCCL	73.77	70.22	61.19	58.07	61.01	64.85±6.75
	MFCCL	83.99	86.18	84.48	83.85	84.81	84.66±0.93
Wiki	DMVCCL	79.07	73.66	74.84	79.05	77.15	**76.75±2.45**
	ILMPFTC	30.83	27.38	26.11	24.37	24.03	26.54±2.75
	F3WGCCL	22.84	20.64	21.98	15.19	15.79	19.29±3.56
	MFCCL	30.83	27.38	26.11	24.37	24.03	26.54±2.75

example, concepts at coarser granularity are less sensitive to missing values, while concepts at finer granularity are more sensitive to missing values. Moreover, the extent of the concept naturally provides a cross-view information that can be utilized to guide the concept learning in incomplete views by leveraging the extents of concepts in complete views. Therefore, exploring the methods for multi-view concept learning for incomplete multi-view data based on these two characteristics is an interesting topic.

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