

Adaptive Granular-Ball Concept-Cognitive Learning for Efficient and Robust Fuzzy Knowledge Representation in Classification Tasks

Doudou Guo ¹, Weihua Xu ¹, Shuyin Xia ¹, Senior Member, IEEE, Weiping Ding ², Senior Member, IEEE, Yuhua Qian ³, Member, IEEE, and Kehua Yuan ⁴

Abstract—In many real-world scenarios, data unreliability and information-processing uncertainty seriously affect the credibility and reliability of artificial intelligence models. Probing a valuable cognitive learning paradigm is crucial for supporting and implementing the proposed artificial intelligence hypothesis. Concept-cognitive learning (CCL), as an emerging cognitive learning technology, investigates the knowledge-learning process using limited data through the lens of concepts. However, the existing CCL research has one prevalent disadvantage: instability and complex cognition. Thus, this work focuses on constructing an effective and interpretable cognitive learning paradigm for acquiring knowledge. Inspired by a multigranularity knowledge representation, this article proposes an adaptive and robust fuzzy knowledge representation paradigm, namely, granular-ball concept-cognitive learning (GB-CCL). Unlike existing CCL systems, GB-CCL is a nonparameter cognitive learning system that achieves more accurate, faster adaptive capabilities for concept learning. The concept acquired through cognitive learning is structured conceptual knowledge, which is naturally interpretable and robust. Finally, extensive experiments verify that GB-CCL is an effective cognitive learning method with significantly better performance than state-of-the-art models in classification and robust learning.

Index Terms—Fuzzy concept, granular computing (GrC), granular-ball computing (GBC), knowledge discovery, concept-cognitive learning (CCL).

I. INTRODUCTION

DESPITE the storm of artificial intelligence that has swept the world and deeply affected the development of all walks

Received 25 January 2026; accepted 9 February 2026. Date of publication 12 February 2026; date of current version 5 May 2026. This work was supported by the National Natural Science Foundation of China under Grant 624B2104 and Grant 62376229. Recommended by Associate Editor R. Jensen. (Corresponding author: Weihua Xu.)

Doudou Guo is with the School of Mathematics, China University of Mining and Technology, Xuzhou 221116, China (e-mail: doudou876517690@126.com).

Weihua Xu is with the College of Artificial Intelligence, Southwest University, Chongqing 400715, China (e-mail: chxuwh@gmail.com).

Shuyin Xia is with the Chongqing Key Laboratory of Computational Intelligence, Chongqing University of Posts and Telecommunications, Chongqing 400065, China (e-mail: xiasy@cqupt.edu.cn).

Weiping Ding is with the School of Artificial Intelligence and Computer Science, Nantong University, Nantong 226019, China (e-mail: dwp9988@163.com).

Yuhua Qian is with the Institute of Big Data Science and Industry, Shanxi University, Taiyuan 030006, China (e-mail: jinchengqyh@126.com).

Kehua Yuan is with the School of Computer Science and Technology, Tongji University, Shanghai 201804, China (e-mail: yuan945527632@163.com).

The source code is publicly available at <https://github.com/G-dataexplorer/GBCL>.

Digital Object Identifier 10.1109/TFUZZ.2026.3664190

of life, there are many tough and common problems when different artificial intelligence methods encounter complex scenarios, such as credibility, reliability, and interpretability. Simulating human cognitive processes is fundamental in artificial intelligence and cognitive computing [1], [2]. In exploring how to effectively cognitive and learning a thing, granular computing (GrC) may be viewed as a novel, interesting, and interpretable theory. One emphasizes observing, analyzing, and understanding things at multiple granularities, perspectives, and levels [3], [4], [5]. Undoubtedly, GrC has become an active research direction in artificial intelligence and cognitive computing.

Granular-ball computing (GBC) and concept-cognitive learning (CCL) are two influential GrC studies on information processing and cognitive learning, with different emphases. According to Chen [6], “human cognition has the characteristics of large-scale priority.” Then, Xia et al. [7] proposed an efficient, robust, and interpretable multigranularity computing paradigm, named GBC. One can depict the distribution characteristics of the sample in the data space and thereby improve the classification performance of rough sets (RSs). Beyond the traditional RSs, the granular ball (GB) has the advantage of being fast and accurate in data analysis. In some sense, the GBC model can simultaneously express both the classical and the neighborhood RSs, enabling it not only to handle continuous data but also to use equivalence classes for knowledge representation. Moreover, extended GBC models have been investigated in many ways to address different problem needs. For instance, Xia et al. [8] utilized the accurate classification advantage of GB to propose the GB K-nearest neighbor (GB-KNN) for enhancing the classification performance of KNN. granular-rectangular rough set (GRRS) is presented in the paper [9] to improve neighborhood RSs by taking advantage of the adaptive learning of GB. Building upon this, Xia et al. [10] further proposed an effective GB generation approach for general classification tasks. Cao et al. [11] investigated GB-based open and continual feature selection in dynamic environments. Li et al. [12] introduced a GB regeneration clustering method grounded in the principle of justifiable granularity.

CCL is the science of cognition and learning things via concepts [13], [14], [15]. As we all know, the concept is the basic unit of thought underlying human intelligence and communication, and the cognitive and learning concepts are important issues for studying artificial intelligence [16], [17]. Currently, CCL, an emerging artificial intelligence method for concept learning and knowledge discovery, has received much attention. For instance,

Li et al. [18] analyzed the concept-learning mechanism from a cognitive psychology principle. Shi et al. [19] established an incremental CCL model for classification tasks. Niu et al. [20] put forward a parallel computing method for CCL. Shi et al. [21] proposed a concurrent CCL method. Zhang et al. [22] studied CCL based on attribute topology. Deng et al. [23] offered a multiconceptual information acquisition for hierarchical classification. Lin et al. [24] analyzed a concept reduction from a global relevance and redundancy viewpoint. With the in-depth study of CCL in knowledge modeling and method application, some researchers are concerned that it can be combined with machine learning to broaden the research horizon of this theory further. In recent papers, Mi et al. proposed a fuzzy-based CCL model [25] and a semisupervised CCL method [26]. Guo and Xu successively put forward a series of CCL models for complex data analysis, such as two-way CCL [27], memory-based CCL [28], and fuzzy-granular CCL [29]. In addition, Wang et al. [30] proposed a multiview CCL framework that integrates high-order information fusion of fuzzy attributes. Liu et al. [31] further explored a stochastic CCL model for missing multilabel learning. In some sense, CCL aims to explore accurate and effective mechanisms for knowledge discovery through cognitive and learning concepts.

Following the no-free-lunch theorem, we find that although both are popular intelligence information-processing approaches, they have unavoidable deficiencies: GBC sacrifices precision for speed, and CCL sacrifices speed for precision. In this sense, a fruitful marriage of the two may be a good choice for knowledge discovery, as it can help eliminate mutual deficiencies. From an extender view of CCL and GB, we built a granular-ball concept cognitive learning (GB-CCL) by introducing the GB, three-way concept analysis to the cognitive concept process for high-efficiency exploring knowledge from complex data. The main idea of this work is three-fold: 1) the GB in a fuzzy decision formal context; 2) the justifiable GB generation for concept learning; and 3) the label prediction based on concept recognition. By doing so, GB-CCL achieves highly accurate classification and significantly outperforms its peers. Furthermore, we summarize the main contributions as follows.

- 1) It proposes a novel and adaptive GB-CCL framework for fuzzy knowledge representation in classification tasks, namely, GB-CCL. Extensive experiments demonstrate the robustness and effectiveness of the proposed GB-CCL method.
- 2) It introduces a fuzzy three-way concept learning mechanism based on GBs in a fuzzy decision formal context to improve the robustness of concept learning, which leverages GBC and three-way concept analysis to enhance the robustness of the GB-CCL.
- 3) It designs a justifiable GB generation model for concept learning to acquire structured conceptual knowledge from complex data. Beyond the traditional CCL, GB-CCL enables fast and adaptive acquisition of structured conceptual knowledge.
- 4) It builds an effective label-prediction method based on the fuzzy three-way concept center in the concept-cognitive process, thereby enhancing the concept recognition performance of fuzzy data.

The rest of this article is organized as follows. Section II revisits several necessary notions for GB-CCL. A GB concept learning mechanism is introduced in Section III. The proposed GB-CCL method is shown in Section IV. In Section V, the experimental analysis is reported. Finally, Section VI concludes this article.

II. PRELIMINARIES

Considering that the proposed GB-CCL is based on the ideas of CCL and GBC, this section briefly reviews some basic knowledge with regard to them. A more detailed description can be found in the corresponding references [10], [27], [32]. As is well known, the standard CCL methods cannot tackle continuous data directly, and the fuzzy-based CCL is a straightforward approach to discovering the knowledge embedded in continuous data. Nevertheless, in the real scenario, many tasks are described with numerical (or fuzzy) data, especially in classification problems. Hence, throughout this article, we will consider a fuzzy formal context to build the model.

A quintuple (G, M, \tilde{I}, D, J) is a regular fuzzy decision formal context, where (G, M, \tilde{I}) and (G, D, J) are, respectively, a conditional formal context and a decision formal context.

- 1) $G = \{x_1, x_2, \dots, x_n\}$ is a nonempty set of objects.
- 2) $M = \{a_1, a_2, \dots, a_m\}$ is a nonempty set of attributes.
- 3) $\tilde{I} : G \times M \rightarrow [0, 1]$ is a fuzzy relation on $G \times M$, where $\tilde{I}(x, a)$ denotes the fuzzy membership degree of x with respect to a .
- 4) $D = \{d_1, d_2, \dots, d_l\}$ is a decision label set, which induces a decision partition $G/D = \{D_1, D_2, \dots, D_l\}$, where $D_i = \{x \in G \mid J(x, d_i) = 1\}$, $i = 1, \dots, l$.
- 5) $J : G \times D \rightarrow \{0, 1\}$ is a binary relation between objects and decision labels.

A. CCL System

Let 2^G and 2^D be two power sets of G and D , Γ^M is the set of all fuzzy sets on M . Suppose that $\tilde{\mathcal{F}} : 2^G \rightarrow \Gamma^M$ and $\mathcal{P} : \Gamma^M \rightarrow 2^G$ are two set-valued mappings. For simplicity, they are rewritten as $\tilde{\mathcal{F}}$ and \mathcal{P} , respectively. For any $X_1, X_2 \subseteq G$, and $\tilde{B} \in \Gamma^M$, the pair $\tilde{\mathcal{F}}$ and \mathcal{P} is called cognitive operators, if the following statements hold:

- 1) $X_1 \subseteq X_2 \Rightarrow \tilde{\mathcal{F}}(X_2) \subseteq \tilde{\mathcal{F}}(X_1)$;
- 2) $\tilde{\mathcal{F}}(X_1 \cup X_2) \subseteq \tilde{\mathcal{F}}(X_1) \cap \tilde{\mathcal{F}}(X_2)$;
- 3) $\mathcal{P}(\tilde{B}) = \{x \in G \mid \tilde{B} \subseteq \tilde{\mathcal{F}}(x)\}$.

Definition 1: Let $FC = (G, M, \tilde{I}, D, J)$ be a fuzzy decision formal context, for any $X \subseteq G$ and $\tilde{B} \in \Gamma^M$, the cognitive operators $\tilde{\mathcal{F}}$ and \mathcal{P} can be defined as follows:

$$\tilde{\mathcal{F}}(X)(a) = \bigwedge_{x \in X} \tilde{I}(x, a), a \in M \quad (1)$$

$$\mathcal{P}(\tilde{B}) = \{x \in G \mid \tilde{B}(a) \leq \tilde{I}(x, a) \quad \forall a \in B\}. \quad (2)$$

Generally speaking, $\tilde{\mathcal{F}}$ and \mathcal{P} are called a pair of positive cognitive operators. A pair (X, \tilde{B}) is called a fuzzy concept if $\tilde{\mathcal{F}}(X) = \tilde{B}$ and $\mathcal{P}(\tilde{B}) = X$, where X and \tilde{B} are referred to as the extent and intent of the fuzzy concept (X, \tilde{B}) . Similarly, the negative cognitive operators can be defined as follows.

Definition 2: Given a fuzzy decision formal context $FC = (G, M, \tilde{I}, D, J)$, for any $X \subseteq G$ and $\tilde{B} \in \Gamma^M$, the negative cognitive operators $\tilde{\mathcal{F}}^- : 2^G \rightarrow \Gamma^M$ and $\mathcal{P}^- : \Gamma^M \rightarrow 2^G$ are defined as

$$\tilde{\mathcal{F}}^-(X)(a) = \bigwedge_{x \in X} \tilde{I}^-(x, a), a \in M \quad (3)$$

$$\mathcal{P}^-(\tilde{B}) = \{x \in G | \tilde{B}(a) \leq \tilde{I}^-(x, a) \quad \forall a \in B\} \quad (4)$$

where $\tilde{I}^- = \{<(x, a), 1 - \tilde{I}(x, a)> | (x, a) \in G \times M\}$ denotes the complement of $\tilde{I}(x, a)$, representing the nonmembership degree of object x with respect to a .

Definitions 1 and 2 show that the positive and negative cognitive operators describe the cognitive process between objects and attributes. Meanwhile, some studies [28], [32] combine these two into a unique cognitive operator for simultaneously expressing positive and negative information.

Definition 3: Given $FC = (G, M, \tilde{I}, D, J)$, for any $X \subseteq G$ and $\tilde{B}_1, \tilde{B}_2 \in \Gamma^M \times \Gamma^M$, the fuzzy three-way concept cognitive operator $\tilde{\mathcal{F}}^\nabla : 2^G \rightarrow \Gamma^M \times \Gamma^M$ and $\mathcal{P}^\nabla : \Gamma^M \times \Gamma^M \rightarrow 2^G$ are defined as

$$\tilde{\mathcal{F}}^\nabla(X) = (\tilde{\mathcal{F}}(X), \tilde{\mathcal{F}}^-(X)) \quad (5)$$

$$\mathcal{P}^\nabla(\tilde{B}_1, \tilde{B}_2) = \mathcal{P}(\tilde{B}_1) \cap \mathcal{P}^-(\tilde{B}_2). \quad (6)$$

Then, $(X, (\tilde{B}_1, \tilde{B}_2))$ is called a fuzzy three-way concept if $\tilde{\mathcal{F}}^\nabla(X) = (\tilde{B}_1, \tilde{B}_2)$ and $\mathcal{P}^\nabla(\tilde{B}_1, \tilde{B}_2) = X$. More details on fuzzy concept or fuzzy three-way concept can be found in [32], [25], and [33].

B. Granular-Ball Computing (GBC)

GBC, an important extension of neighborhood RSs, provides a new multigranularity computing framework for characterizing data distribution using a pair of radius and center. Thus, it is necessary to introduce the notion of the neighborhood granular in this section.

Given $FC = (G, M, \tilde{I}, D, J)$, for any $x \in G$ and $\tilde{B} \in \Gamma^M$, the neighborhood granular of x with respect to attribute subset $B \subseteq M$ is defined as

$$\delta_{\tilde{B}}(x) = \{x' \in G | \text{dis}(x, x') \leq \delta\} \quad (7)$$

where $\text{dis}(x, x') = \sqrt{\sum_{a \in \tilde{B}} \|\tilde{I}(x, a) - \tilde{I}(x', a)\|^2}$, $\|\cdot\|$ denotes the 2-norm, and $\delta > 0$ is a distance threshold.

Although the neighborhood relationship can be defined by a radius that captures the similarity between any two samples, selecting a radius is also a significant challenge, particularly in terms of flexibility and generalization. A GB, however, can overcome this challenge by defining a center and a radius.

Compared with the neighborhood granular, the significant advantage of GBC is that each granule can be generated adaptively based on the data distribution, and the radius does not need to be set in advance. The radius equals the average distance from all instances in the GB to its center, and the radius can also be set as the maximum distance. Hence, a GB with a center and radius can be used as input to the learning strategy or as precise measures to represent the sample space in a fuzzy decision formal context, thereby achieving multigranularity cognitive computing characteristics and accurately describing the instance space of data. Besides, GB generation is another important issue of GBC. As

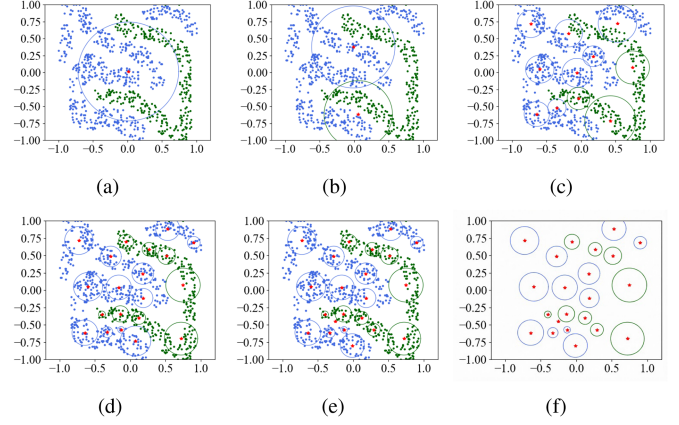


Fig. 1. GB generation process in [10], where (a)–(f) depicts the detailed split process.

analyzed in paper [10], GB generation can be incorporated into GBC to enhance the classification performance. A detailed GB generation process can be found in Fig. 1.

III. GB CONCEPT LEARNING

This section presents new notions and the related theoretical analysis of the GB concept in fuzzy decision formal contexts. Motivated by GBC and CCL, we offer a GB concept learning model that integrates the merits of GB, fuzzy three-way concept, and concept learning.

A. GB in Fuzzy Decision Formal Context

Generally speaking, fuzzy concept learning is more dependent on initial information, such as a granular concept learning based on object-oriented information was offered in [25], a progressive fuzzy concept according to neighborhood information granular was proposed in [32], and a fuzzy concept learning via cosine similarity granular was studied in [28]. Meanwhile, to enhance the effectiveness of GB-CCL, this section primarily defines a novel information granule, i.e., the GB. Building on this thought, we present a GB concept learning for the subsequent concept-cognitive process, which can be described as follows.

Definition 4: Let $GB_s = \{GB_1, GB_2, \dots, GB_u\}$ be a set of GBs. For any $GB_i^{D_k}$ associated with D_k , its center c_i , radius r_i , and purity $Pur(GB_i^{D_k})$ are defined as follows:

$$GB_i^{D_k} = \{x \in G | \sqrt{\|\tilde{\mathcal{F}}(x) - c_i\|^2} \leq r_i\} \quad (8)$$

$$c_i = \frac{1}{|GB_i^{D_k}|} \sum_{x \in GB_i^{D_k}} \tilde{\mathcal{F}}(x) \quad (9)$$

$$r_i = \frac{1}{|GB_i^{D_k}|} \sum_{x \in GB_i^{D_k}} \sqrt{\|\tilde{\mathcal{F}}(x) - c_i\|^2} \quad (10)$$

$$Pur(GB_i^{D_k}) = \frac{\max_{1 \leq j \leq k} |GB_i^{D_k} \cap D_j|}{|GB_i^{D_k}|} \quad (11)$$

where $|\cdot|$ denote the cardinality of a set.

Definition 5: Let $\mathcal{S}^{D_k} = \{GB_1^{D_k}, GB_2^{D_k}, \dots, GB_s^{D_k}\}$ be a set of GBs associated with D_k , where $s = |\mathcal{S}^{D_k}|$. We say that \mathcal{S}^{D_k} is a GB subspace of D_k if the following properties hold.

- 1) $\forall GB_i^{D_k} \in \mathcal{S}^{D_k}, GB_i^{D_k} \neq \emptyset$.
- 2) $\forall GB_i^{D_k} \in \mathcal{S}^{D_k}, GB_i^{D_k} \subseteq D_k$.
- 3) $\bigcup_{i=1}^s GB_i^{D_k} = D_k$.
- 4) $\min |\mathcal{S}^{D_k}|$, s.t. items 1) and 2) hold.

As indicated in Definitions 4 and 5, we only consider the basic form of GB for a fuzzy decision formal context, and a more concrete agenda for GB generation is presented in Section IV. Furthermore, as discussed above, we can learn concepts using the GB and cognitive operators.

B. Fuzzy Three-Way Concept Based on GB

As analyzed in [27], [28], and [32], the effectiveness of the three-way concept lies in its emphasis on simultaneously characterizing information from both positive and negative perspectives, so three-way concept analysis can be incorporated into a CCL model to improve its accuracy in describing cognitive processes, especially in a fuzzy environment. Following this principle, we incorporate the GB into the concept learning with the fuzzy three-way concept to model a novel fuzzy concept learning mechanism.

Theorem 1: Given $FC = (G, M, \tilde{I}, D, J)$ and $\mathcal{S}^{D_k} = \{GB_1^{D_k}, GB_2^{D_k}, \dots, GB_s^{D_k}\}$, $s = 1, 2, \dots, |D_k|$ with $G/D = \{D_1, D_2, \dots, D_l\}$ on FC . For any $GB_i^{D_k} \subseteq D_k$, $(\mathcal{P}\tilde{\mathcal{F}}(GB_1^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_1^{D_k}), (\tilde{\mathcal{F}}(GB_1^{D_k}), \tilde{\mathcal{F}}^-(GB_1^{D_k})))$ is a fuzzy three-way concept.

Proof: To prove this theorem, one can divide it into the following two steps:

- 1) prove $\mathcal{P}^\nabla(\tilde{\mathcal{F}}(GB_i^{D_k}), \tilde{\mathcal{F}}^-(GB_i^{D_k})) = \mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k})$ is valid;
- 2) prove $\tilde{\mathcal{F}}^\nabla(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k})) = (\tilde{\mathcal{F}}(GB_i^{D_k}), \tilde{\mathcal{F}}^-(GB_i^{D_k}))$ is valid.
 - a) For 1), according to definition 3, we have

$$\begin{aligned} & \mathcal{P}^\nabla(\tilde{\mathcal{F}}(GB_i^{D_k}), \tilde{\mathcal{F}}^-(GB_i^{D_k})) \\ &= \mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}). \end{aligned}$$

- b) For 2), according to definition 2, we further obtain
$$\begin{aligned} & \mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}) = \{y \in D_k | \tilde{I}(y, a) \geq \bigwedge_{o \in GB_i^{D_k}} \tilde{I}(o, a), a \in M\} \cap \{y \in D_k | \tilde{I}^-(y, a) \geq \bigwedge_{o \in GB_i^{D_k}} \tilde{I}^-(o, a), a \in M\} \\ &= \{y \in D_k | \tilde{I}(y, a) \geq \bigwedge_{o \in GB_i^{D_k}} \tilde{I}(o, a), a \in M\} \cap \{y \in D_k | 1 - \tilde{I}^-(y, a) \leq 1 - \bigwedge_{o \in GB_i^{D_k}} \tilde{I}^-(o, a), a \in M\} \\ &= \{y \in D_k | \tilde{I}(y, a) \geq \bigwedge_{o \in GB_i^{D_k}} \tilde{I}(o, a), a \in M\} \cap \{y \in D_k | \tilde{I}(y, a) \leq \bigvee_{o \in GB_i^{D_k}} \tilde{I}(o, a), a \in M\} \\ &= \{y \in D_k | \bigwedge_{o \in GB_i^{D_k}} \tilde{I}(o, a) \leq \tilde{I}(y, a) \leq \bigvee_{o \in GB_i^{D_k}} \tilde{I}(o, a), a \in M\}. \end{aligned}$$

Thus, for any $a \in M$, we have

$$\begin{aligned} & \tilde{\mathcal{F}}(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}))(a) = \\ & \bigwedge_{o \in GB_i^{D_k}} \tilde{I}(o, a) = \tilde{\mathcal{F}}(GB_i^{D_k})(a). \end{aligned}$$

Meanwhile, for any $a \in M$, we obtain

$$\begin{aligned} & \mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}) = \{y \in G_i | \bigwedge_{o \in GB_i^{D_k}} \tilde{I}^-(o, a)\} \leq \tilde{I}^-(y, a) \leq \\ & \bigvee_{o \in GB_i^{D_k}} \tilde{I}^-(o, a), a \in M\}, \tilde{\mathcal{F}}^-(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}))(a) = \bigwedge_{o \in GB_i^{D_k}} \tilde{I}^-(o, a) = \\ & \tilde{\mathcal{F}}^-(GB_i^{D_k})(a). \end{aligned}$$

Hence

$$\begin{aligned} & \tilde{\mathcal{F}}^\nabla(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k})) \\ &= (\tilde{\mathcal{F}}(GB_i^{D_k}), \tilde{\mathcal{F}}^-(GB_i^{D_k})). \end{aligned}$$

By combining 1) and 2), this theorem is proven.

Based on Theorem 1, we can obtain structured conceptual knowledge by defining a new fuzzy concept, the GB, from both positive and negative perspectives. In this way, the fuzzy three-way concepts learned have the advantage of being more robust and precise than the way in [13], [25], [28], and [32]. Then, we can build the corresponding fuzzy three-way concept space as follows.

Definition 6: Let $\mathcal{S}^{D_k} = \{GB_1^{D_k}, GB_2^{D_k}, \dots, GB_s^{D_k}\}$ ($s = 1, 2, \dots, |D_k|$) be a GB subspace on FC . The fuzzy three-way concept subspace $\tilde{\mathcal{G}}^{D_k}$ based on GBs is defined as

$$\begin{aligned} \tilde{\mathcal{G}}^{D_k} = & \{(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}), (\tilde{\mathcal{F}}(GB_i^{D_k}), \\ & \tilde{\mathcal{F}}^-(GB_i^{D_k}))) | GB_i^{D_k} \in \mathcal{S}^{D_k}\}. \end{aligned} \quad (12)$$

Theorem 2: Let \mathcal{S}^{D_k} be a GB subspace of D_k , and $\tilde{\mathcal{G}}^{D_k}$ be the corresponding fuzzy three-way concept subspace constructed from \mathcal{S}^{D_k} . Then, the following property holds:

$$1 \leq |\tilde{\mathcal{G}}^{D_k}| \leq |\mathcal{S}^{D_k}|. \quad (13)$$

Proof: The proof can be divided into the following four cases.

- 1) If $|\mathcal{S}^{D_k}| = 1$, i.e., D_k contains only one GB, then the fuzzy three-way concept subspace generated from it has exactly one concept. Thus, $|\tilde{\mathcal{G}}^{D_k}| = 1$.
- 2) If $|\mathcal{S}^{D_k}| > 1$, i.e., D_k contains multiple GBs and for each $x \in D_k$, there exists at least one $GB_i^{D_k} \in \mathcal{S}^{D_k}$ such that $x \in GB_i^{D_k}$, then at least two concepts will be generated. Hence, $|\tilde{\mathcal{G}}^{D_k}| \geq 1$.
- 3) If exist $GB_i^{D_k}, GB_j^{D_k} \in \mathcal{S}^{D_k}$ such that $(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}), (\tilde{\mathcal{F}}(GB_j^{D_k}), \tilde{\mathcal{F}}^-(GB_j^{D_k}))) = (\mathcal{P}\tilde{\mathcal{F}}(GB_j^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_j^{D_k}), (\tilde{\mathcal{F}}(GB_j^{D_k}), \tilde{\mathcal{F}}^-(GB_j^{D_k})))$, then two different GBs generate the same fuzzy three-way concept, and thus $|\tilde{\mathcal{G}}^{D_k}| < |\mathcal{S}^{D_k}|$.
- 4) If for all $GB_i^{D_k}, GB_j^{D_k} \in \mathcal{S}^{D_k}$ with $i \neq j$, their corresponding concepts are distinct, then each GB generates a unique concept, giving $|\tilde{\mathcal{G}}^{D_k}| = |\mathcal{S}^{D_k}|$.

By combining 1)–4) this theorem is proven.

By Theorem 1 and definition 6, we obtain the fuzzy three-way concept based on GB via two pair of cognitive operators (i.e., $(\tilde{\mathcal{F}}, \mathcal{P})$ and $(\tilde{\mathcal{F}}^-, \mathcal{P}^-)$), and then $\tilde{\mathcal{G}} = \{\tilde{\mathcal{G}}^{D_1}, \tilde{\mathcal{G}}^{D_2}, \dots, \tilde{\mathcal{G}}^{D_l}\}$ is established, in which the GB is the initial information granular for fuzzy three-way concept learning. Unlike [27], [28], and [32], using GB as information cues can characterize data distributions faster and has the advantage of no parameters.

IV. PROPOSED GB-CCL METHOD

Based on the above discussion, this section first presents the proposed GB-CCL, which uses GBs for initial concept learning. Then, it discusses a new label-prediction method within the concept-cognitive process. Furthermore, we also summarize the overall procedure for GB-CCL.

A. Justifiable GB for Concept Learning

Definitions 4 and 5 only give the general form of the GB and GB subspace and do not discuss how to generate a justifiable GB and its GB space in a fuzzy decision formal context. Thus, this section is necessary to propose an initial GB generation for concept learning in a fuzzy decision formal context.

Given a fuzzy decision formal context $FC = (G, M, \tilde{I}, D, J)$, where $G/D = \{D_1, D_2, \dots, D_l\}$, the justifiable GB generation method consists of four parts: initialization, update of granular divisions, GB generation, and concept learning.

1) *Initialization*: Randomly select l objects in different decision subspaces as the initial granular center set $\hat{C}_0 = \{c_1, c_2, \dots, c_l\} (c_i \in D_i)$. For each center c_i , define the initial granular division as

$$\text{div}_0(c_i) = \{x \in G \mid c_i = \arg \min_{c_j \in \hat{C}_0} \|x - c_j\|\}. \quad (14)$$

Each $\text{div}_0(c_i)$ is called an initial granular division of G .

2) *Update granular division*: At the w th granular division, for any $\text{div}_{w-1}(c_i), i = 1, 2, \dots, l$ in the last granular division process. If $\text{div}_{w-1}(c_i)$ is *pure* (all objects have the same decision label), then stop dividing: $\text{Div}_w^i = \text{div}_{w-1}(c_i)$. Otherwise, $\text{div}_{w-1}(c_i)$ is *impure* with h different classes. Keep the original center c_i and randomly select $h - 1$ objects from other classes as new centers $\hat{C}_w^i = \{c_1, \dots, c_h\}$. Update the granular division as

$$\text{div}_w(c_j) = \{x \in \text{div}_{w-1}(c_i) \mid c_j = \arg \min_{c_k \in \hat{C}_w^i} \|x - c_k\|\} \\ j = 1, 2, \dots, h. \quad (15)$$

Repeat until all granular divisions are pure or satisfy a given purity threshold.

3) *GB generation*: Let the final granular centers and corresponding granular division are $\hat{C} = \{c_1, c_2, \dots, c_f\}$ and $\text{Div} = \{\text{div}(c_1), \text{div}(c_2), \dots, \text{div}(c_f)\}$, respectively. The decision label of each granular division is given by $\text{DivLab} = \{D(c_1), D(c_2), \dots, D(c_f)\}$. Then, the GBs induced by Div are generated based on the granular centers and radius, which could be described as follows:

$$r_i = \frac{1}{|\text{div}(c_i)|} \sqrt{\sum_{x \in \text{div}(c_i)} \|x - c_i\|^2} \quad (16)$$

$$\text{GB}_i^{D(c_i)} = \{x \in D(c_i) \mid \text{dis}(x, c_i) \leq r_i\}. \quad (17)$$

Then, the quadruple $(c_i, r_i, \text{GB}_i^{D(c_i)}, D(c_i))$ is called a justifiable GB, and the justifiable GB space is $\mathcal{B} = \{(c_i, r_i, \text{GB}_i^{D(c_i)}, D(c_i)), i = 1, 2, \dots, f\}$.

4) *Concept learning*: The GB concept learning could be characterized based on the above justifiable GB generation. For any $(c_i, r_i, \text{GB}_i^{D(c_i)}, D(c_i)) \in \mathcal{B}$, the fuzzy three-way concept

Algorithm 1: Justifiable GB Generation for Concept Learning.

Require: A fuzzy decision formal context

$$FC = (G, M, \tilde{I}, D, J).$$

Ensure: Granular-ball space \mathcal{B} , fuzzy three-way concept space $\tilde{\mathcal{G}}$.

- 1: Initialize $\mathcal{B} = \emptyset, \tilde{\mathcal{G}} = \emptyset$;
- 2: **for** each $D_k \in G/D$ **do**
- 3: Initialize $\tilde{\mathcal{G}}^{D_k} = \emptyset, \text{Div}_0 = \emptyset$;
- 4: Randomly select one object from each decision class in G/D as the initial granular center set $\hat{C}_0 = \{c_1, c_2, \dots, c_{|G/D|}\}, c_i \in D_i$
- 5: **for all** $c_i \in \hat{C}_0$
- 6: Compute $\text{div}_0(c_i)$ according to (14);
- 7: **if** $\text{div}_0(c_i) \subseteq D_k$ **then**
- 8: $\text{Div}_0 \leftarrow \text{Div}_0 \cup \{\text{div}_0(c_i)\}$;
- 9: **else**
- 10: Update $\text{div}_0(c_i)$ according to (15);
- 11: **end if**
- 12: **end for**
- 13: **for all** $\text{div}(c_i) \in \text{Div}_0$
- 14: Compute the granular-ball radius r_i according to (16);
- 15: Generate the granular ball $\text{GB}_i^{D(c_i)}$ according to (17);
- 16: $\mathcal{B} \leftarrow \mathcal{B} \cup \{(c_i, r_i, \text{GB}_i^{D(c_i)}, D(c_i))\}$;
- 17: **end for**
- 18: **end for**
- 19: **for all** $\text{GB}_i^{D(c_i)} \in \mathcal{B}$ with $D(c_i) = D_k$
- 20: Construct a fuzzy three-way concept $(X, (\tilde{A}_1, \tilde{A}_2))$ according to (18) and (19);
- 21: Construct a fuzzy three-way concept subspace $\tilde{\mathcal{G}}^{D_k}$ according to (20);
- 22: $\tilde{\mathcal{G}}^{D_k} \leftarrow \tilde{\mathcal{G}}^{D_k} \cup \{(X, (\tilde{A}_1, \tilde{A}_2))\}$;
- 23: **end for**
- 24: $\tilde{\mathcal{G}} \leftarrow \tilde{\mathcal{G}} \cup \{\tilde{\mathcal{G}}^{D_k}\}$;
- 25: **return** \mathcal{B} and $\tilde{\mathcal{G}}$.

on this GB is defined as follows:

$$\tilde{\mathcal{F}}^\nabla(\text{GB}_i^{D(c_i)}) = (\tilde{\mathcal{F}}(\text{GB}_i^{D(c_i)}), \tilde{\mathcal{F}}^-(\text{GB}_i^{D(c_i)})) \quad (18)$$

$$\mathcal{P}^\nabla(\tilde{\mathcal{F}}(\text{GB}_i^{D(c_i)}), \tilde{\mathcal{F}}^-(\text{GB}_i^{D(c_i)})) = \mathcal{P}(\tilde{\mathcal{F}}(\text{GB}_i^{D(c_i)})) \cap \mathcal{P}^-(\tilde{\mathcal{F}}^-(\text{GB}_i^{D(c_i)})). \quad (19)$$

Then, its fuzzy three-way concept subspace $\tilde{\mathcal{G}}^{D_k}$ can be constructed as follows:

$$\tilde{\mathcal{G}}^{D_k} = \{(\mathcal{P}\tilde{\mathcal{F}}(\text{GB}_i^{D(c_i)}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(\text{GB}_i^{D(c_i)})) \\ (\tilde{\mathcal{F}}(\text{GB}_i^{D(c_i)})\tilde{\mathcal{F}}^-(\text{GB}_i^{D(c_i)})) \mid \text{GB}_i^{D(c_i)} \in D_k\}. \quad (20)$$

Finally, the complete fuzzy three-way concept space is $\tilde{\mathcal{G}} = \{\tilde{\mathcal{G}}^{D_1}, \tilde{\mathcal{G}}^{D_2}, \dots, \tilde{\mathcal{G}}^{D_k}\}$.

Remark: Compared with existing GB methods, this justifiable GB generation approach effectively removes boundary and noisy samples, and leverages the resulting GB clusters as adaptive information granules for fuzzy three-way concept learning.

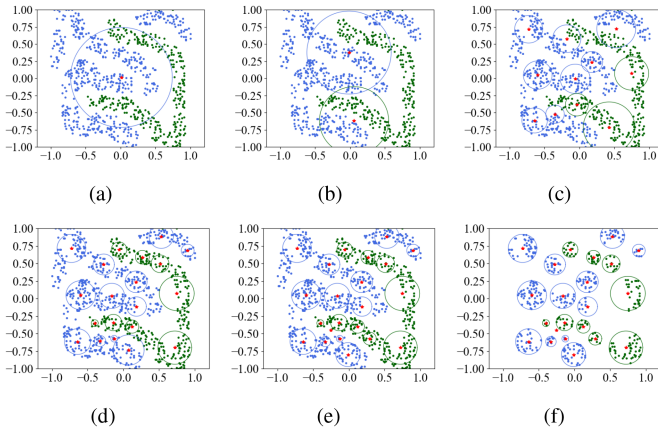


Fig. 2. Justifiable GB generation process. (a)–(f) Splitting process at different steps. Compared with the existing approaches that utilize class centers as representatives for subsequent learning in [10], the proposed method removes boundary and noisy samples and leverages the resulting GB clusters as adaptive information granules for concept learning.

After the above four steps, we obtain the justifiable GB generation method for concept learning. Note that $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \dots, f\}$ and $\tilde{\mathcal{G}} = \{\tilde{\mathcal{G}}^{D_1}, \tilde{\mathcal{G}}^{D_2}, \dots, \tilde{\mathcal{G}}^{D_l}\}$ are necessary elements in label prediction at the concept-cognitive processes. A diagram of the GB generation process is shown in Fig. 2.

The time complexity of the GB generation phase (Lines 2–18) mainly depends on the number of decision classes l , the refinement iterations t , the number of objects n , and the feature dimensionality m . Since each refinement involves distance or similarity computations in the m -dimensional space and scans the involved objects at most once, this phase requires $O((l-1)tnm)$ time in the worst case. The justifiable GB representation further introduces an additional cost of $O(km)$ by computing the center and radius of the k GBs. In the fuzzy three-way concept learning stage (Lines 19–24), each object is processed only within its corresponding GB, leading to an intent learning cost of $O(nm)$. The context learning incurs $O(knm)$ due to comparisons between k GBs and n objects. By retaining the dominant terms, the overall worst-case time complexity of Algorithm 1 is $O(((l-1)t+k)nm)$.

B. Label Prediction in Concept-Cognitive Process

In CCL, we strive to develop an effective conceptual model with satisfactory classification performance, yet the actual situation is often less than ideal. Hence, pseudoconcepts [25] and progressive concepts [13], [32] are a great way to recognize fuzzy concept ontologies. Effective concept recognition requires concepts with high representation. Consequently, a label-prediction strategy is proposed in this section to identify objects effectively.

Definition 7: Let $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \dots, f\}$ be a GB subspace, and $(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D(c_i)}) \cap \mathcal{P}\tilde{\mathcal{F}}^-(GB_i^{D(c_i)}), (\tilde{\mathcal{F}}(GB_i^{D(c_i)}), \tilde{\mathcal{F}}^-(GB_i^{D(c_i)})))$ be the fuzzy three-way concept induced by $GB_i^{D(c_i)}$. Then, its

Algorithm 2: Label Prediction in Concept-Cognitive Process.

Require: Fuzzy three-way concept space

$\tilde{\mathcal{G}} = \{\tilde{\mathcal{G}}^{D_1}, \tilde{\mathcal{G}}^{D_2}, \dots, \tilde{\mathcal{G}}^{D_l}\}$, newly input object set $\Delta\mathcal{G}$

Ensure: \mathcal{R}_{o, D_k}^* and predicted label D_k^*

- 1: **for all** $o \in \Delta\mathcal{G}$
- 2: Obtain $(\tilde{\mathcal{F}}^+(o), \tilde{\mathcal{F}}^-(o))$;
- 3: **for all** $\tilde{\mathcal{G}}^{D_k} \in \tilde{\mathcal{G}}$
- 4: Initialize $\mathcal{R}_{o, D_k}^* \leftarrow +\infty$
- 5: **for all** $GB_i^{D(c_i)} \in \tilde{\mathcal{G}}^{D_k}$
- 6: Compute positive center $C_{GB_i^{D(c_i)}}^+$ and negative center $C_{GB_i^{D(c_i)}}^-$ according to (21) and (22);
- 7: Compute concept recognition degree:

$$\mathcal{R}_{o, GB_i^{D(c_i)}} = \sqrt{\|\tilde{\mathcal{F}}^+(o) - C_{GB_i^{D(c_i)}}^+\|^2 + \|\tilde{\mathcal{F}}^-(o) - C_{GB_i^{D(c_i)}}^-\|^2}$$
- 8: **if** $\mathcal{R}_{o, GB_i^{D(c_i)}} < \mathcal{R}_{o, D_k}^*$ **then**
- 9: $\mathcal{R}_{o, D_k}^* \leftarrow \mathcal{R}_{o, GB_i^{D(c_i)}};$
- 10: **end if**
- 11: **end for**
- 12: **end for**
- 13: Predicted label: $D_k^* \leftarrow \arg \min_{D_k} \mathcal{R}_{o, D_k}^*;$
- 14: **end for**
- 15: **return** \mathcal{R}_{o, D_k}^* and D_k^* .

positive and negative centers are defined as follows:

$$C_{GB_i^{D(c_i)}}^+ = \frac{\sum_{x \in \mathcal{P}^\nabla(\tilde{\mathcal{F}}(GB_i^{D(c_i)}), \tilde{\mathcal{F}}^-(GB_i^{D(c_i)}))} \tilde{\mathcal{F}}^+(x)}{|\mathcal{P}^\nabla(\tilde{\mathcal{F}}(GB_i^{D(c_i)}), \tilde{\mathcal{F}}^-(GB_i^{D(c_i)}))|} \quad (21)$$

$$C_{GB_i^{D(c_i)}}^- = \frac{\sum_{x \in \mathcal{P}^\nabla(\tilde{\mathcal{F}}(GB_i^{D(c_i)}), \tilde{\mathcal{F}}^-(GB_i^{D(c_i)}))} \tilde{\mathcal{F}}^-(x)}{|\mathcal{P}^\nabla(\tilde{\mathcal{F}}(GB_i^{D(c_i)}), \tilde{\mathcal{F}}^-(GB_i^{D(c_i)}))|} \quad (22)$$

where $C_{GB_i^{D(c_i)}}^+$ and $C_{GB_i^{D(c_i)}}^-$ denote the positive and negative centers of the fuzzy three-way concept, respectively.

Definition 8: Let o be a newly input object with fuzzy membership degree $\tilde{\mathcal{F}}^+(o)$ and nonmembership degree $\tilde{\mathcal{F}}^-(o)$. For any fuzzy three-way concept induced by $GB_i^{D(c_i)}$, the fuzzy three-way concept recognition degree is defined as

$$\mathcal{R}_{o, GB_i^{D(c_i)}} = \sqrt{\|\tilde{\mathcal{F}}^+(o) - C_{GB_i^{D(c_i)}}^+\|^2 + \|\tilde{\mathcal{F}}^-(o) - C_{GB_i^{D(c_i)}}^-\|^2}$$

where $\|\cdot\|$ denotes the 2-norm. This measure quantifies the distance of object o from the positive and negative centers of the fuzzy three-way concept.

Intrinsically, the recognition value $\mathcal{R}_{o, D(c_i)}$ of a fuzzy three-way concept reflects the similarity between the two concepts. In other words, the greater the value of \mathcal{R} , the smaller the relationship between the two concepts. Thus, for any newly input object o characterized from positive and negative information, we can calculate the concept recognition between $(o, (\tilde{\mathcal{F}}^+(o), \tilde{\mathcal{F}}^-(o)))$

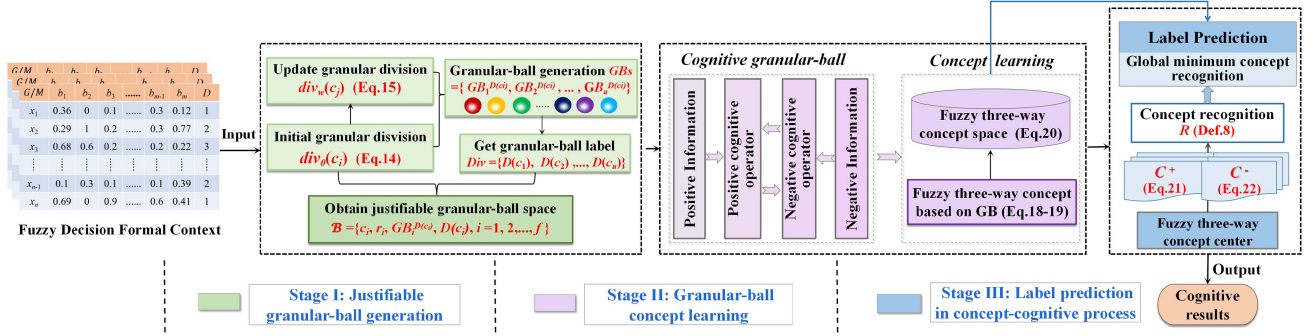


Fig. 3. Overall procedure of the proposed GB-CCL.

and any concept in the fuzzy three-way concept space according to Definition 8. Likewise, we can also calculate a global minimum concept recognition by pairwise comparison and then predict its label, denoted by $D_k^* = \arg \min_{D_k} \mathcal{R}_{o, D(c_i)}$. The detailed procedure is presented in Algorithm 2. In the concept center computation at Line 6, each object contributes to only one corresponding concept center, resulting in a time complexity of $O(nm)$. In addition, the concept recognition process (Lines 7–8) compares the newly added objects in ΔG with the existing k concepts under m features, leading to a complexity of $O(k|\Delta G|m)$. Therefore, the overall worst-case time complexity of Algorithm 2 is $O((n+k|\Delta G|)m)$. By combining Algorithms 1 and 2, the total worst-case time complexity of the proposed method is $O(((l-1)t+k)n+k|\Delta G|m)$.

C. Overall Procedure

As illustrated in Fig. 3, the overall procedure of GB-CCL consists of three stages: 1) justifiable GB generation; 2) GB concept learning; and 3) label prediction in the concept-cognitive process. From the discussions mentioned above, a justifiable GB space $\mathcal{B} = \{(c_i, r_i, GB_i^{D(c_i)}, D(c_i)), i = 1, 2, \dots, f\}$ generation is a vital step in GB-CCL. In this process, we characterize a GB adaptive learning method based on decision division G/D , where information granulation is achieved by splitting granular divisions from large to small, resulting in a generated GB that exhibits fast and accurate characterization of data distribution.

Furthermore, we make full use of the justifiable GB learned in the first stage to carry out concept learning, in which a pair of positive and negative cognitive operators (i.e., $(\tilde{\mathcal{F}}^+, \mathcal{P}^+)$, $(\tilde{\mathcal{F}}^-, \mathcal{P}^-)$) are used to distribute knowledge acquisition from two perspectives of positive information and negative information so as to obtain structured conceptual knowledge, i.e., acquire fuzzy three-way concept based on GB and the corresponding fuzzy three-way concept space (i.e., $\tilde{\mathcal{G}}^{D_k} = \{(\mathcal{P}\tilde{\mathcal{F}}(GB_i^{D_k}) \cap \mathcal{P}^-\tilde{\mathcal{F}}^-(GB_i^{D_k}), (\tilde{\mathcal{F}}(GB_i^{D_k}), \tilde{\mathcal{F}}^-(GB_i^{D_k}))) | GB_i^{D_k} \in \mathcal{S}^{D_k}\}$) for concept recognition in the subsequent stage.

Finally, we design a concept recognition mechanism based on a concept center (i.e., $C_{GB_i^{D(c_i)}}^+$ and $C_{GB_i^{D(c_i)}}^-$) for label prediction in the concept-cognitive process. In this stage, we can describe its positive and negative information (i.e., $(\tilde{\mathcal{F}}^+(o), \tilde{\mathcal{F}}^-(o))$) and then identify it with the concept in the fuzzy three-way concept space $\tilde{\mathcal{G}} = \{\tilde{\mathcal{G}}^{D_1}, \tilde{\mathcal{G}}^{D_2}, \dots, \tilde{\mathcal{G}}^{D_k}\}$,

TABLE I
BASIC DESCRIPTION OF 15 SELECTED DATASETS

| No. | Dataset | | Sample Feature Class | | | Area |
|-----|-------------------------|--------|----------------------|------|----|------------------|
| | Full Name | Abbre. | | | | |
| D1 | Cardiotography | Card | 2126 | 21 | 3 | Life Science |
| D2 | Dermatology | Derma | 366 | 35 | 6 | Life Science |
| D3 | Dry Bean | DryB | 13611 | 17 | 6 | Computer Science |
| D4 | FrogsMFCCs | Frogs | 7195 | 23 | 60 | Other field |
| D5 | Magic | Magic | 19020 | 11 | 2 | Physical Science |
| D6 | Mice Protein Expression | MiPE | 1077 | 69 | 8 | Life Science |
| D7 | Movement | Move | 360 | 91 | 12 | Other field |
| D8 | Parkinsons | Park | 195 | 23 | 2 | Life Science |
| D9 | Seeds | Seeds | 210 | 8 | 3 | Life Science |
| D10 | Segmentation | Segm | 2310 | 19 | 7 | Other field |
| D11 | Wine Quality White | WiQW | 4898 | 12 | 11 | Business |
| D12 | Colon | Colo | 62 | 2000 | 2 | Life Science |
| D13 | Leukemia | Leuk | 72 | 7130 | 2 | Life Science |
| D14 | Lung | Lung | 203 | 3312 | 5 | Life Science |
| D15 | Yale | Yale | 165 | 1025 | 15 | Computer Science |

where the label of the concept with the smallest recognition degree is our prediction label for the new object.

V. EXPERIMENT ANALYSIS

In this section, we design various numerical comparative experiments compared with other approaches on the public datasets. In the subsequent experiments, we aim to answer the following research questions (Rqs): *Rq1*: Does the proposed GB-CCL method outperform related state-of-the-art peers? *Rq2*: What are the differences between GB-CCL, GB, and other CCL mechanisms? *Rq3*: How does the new viewpoint influence the robustness of GB-CCL?

A. General Settings

1) *Datasets*: To evaluate the performance of our GB-CCL model, we chose 15 public datasets from the UCI Machine Learning Repository¹ to make comparisons, and detailed information about them is shown in Table I. Meanwhile, all the experimental datasets are fuzzified into the interval $[0,1]$, and they are first fuzzified according to the following equation:

$$\tilde{I}(x, a) = \frac{f(x, a) - \min(f(a))}{\max(f(a)) - \min(f(a))}$$

¹[Online]. Available: <http://archive.ics.uci.edu/ml/datasets.php>

TABLE II
CLASSIFICATION ACCURACY(%) COMPARISON WITH OTHER 11 POPULAR METHODS ON 15 DATASETS

| No.s | MEEFI | FENN | ssODM | RF | WFCCL | M-FCCL | R-FCCL | GB-KNN | CM-CCL | GBCT | GB-CCLF | GB-CCL |
|------|-------------|-------------|-------------|--------------------|-------------|-------------|--------------------|-------------|-------------|-------------|-------------|---------------------|
| D1 | 80.76±1.91 | 80.76±2.70 | 46.90±3.39 | 76.71±4.27 | 59.08±3.52 | 41.77±4.11 | 76.43±1.41 | 67.59±2.77 | 77.28±2.86 | 52.35±0.33 | 78.60±2.35 | 81.52 ±2.34 |
| D2 | 94.55±2.85 | 84.68±5.70 | 30.59±4.70 | 89.34±5.54 | 91.53±3.91 | 91.33±3.35 | 93.29±4.60 | 71.01±9.69 | 89.10±6.87 | 55.59±0.40 | 95.10±3.58 | 95.65 ±3.18 |
| D3 | 13.79±1.21 | 71.93±0.96 | 14.89±0.60 | 88.41±0.82 | 67.18±4.89 | 48.08±1.49 | 91.60 ±0.73 | 86.72±1.58 | 65.46±3.25 | 38.59±1.87 | 85.45±0.95 | 86.72±1.01 |
| D4 | 15.37±6.44 | 86.39±1.52 | 0.72±0.32 | 0.85±0.27 | 54.84±3.21 | 44.74±2.50 | 81.03±1.94 | 64.17±2.77 | 73.91±2.22 | 11.01±0.00 | 86.27±1.02 | 86.39 ±0.96 |
| D5 | 35.16±0.82 | 80.80±0.76 | 64.84±0.82 | 82.78 ±0.75 | 0.07±0.00 | 73.20±1.35 | 81.64±0.81 | 72.94±1.64 | 74.20±0.68 | 63.02±3.19 | 76.39±0.76 | 75.80±0.96 |
| D6 | 32.60±9.84 | 93.78±2.36 | 13.92±2.36 | 76.60±4.03 | 33.53±6.70 | 40.02±3.95 | 97.49±1.33 | 66.68±7.85 | 90.81±1.40 | 19.50±4.44 | 97.68±1.97 | 97.96 ±1.56 |
| D7 | 11.11±4.54 | 69.17±8.12 | 6.67±2.99 | 6.11±3.41 | 39.44±8.16 | 48.33±9.82 | 48.33±12.23 | 70.83±8.41 | 60.83±6.73 | 16.94±9.82 | 81.94±3.53 | 83.89 ±4.86 |
| D8 | 57.13±18.28 | 84.05±5.74 | 75.32±10.19 | 83.08±6.40 | 53.34±16.9 | 44.16±12.38 | 44.16±12.38 | 85.08±11.64 | 85.11±8.46 | 84.61±0.11 | 91.37±7.89 | 93.26 ±6.57 |
| D9 | 45.71±16.37 | 89.05±3.21 | 33.33±14.02 | 92.38±5.59 | 86.19±7.92 | 90.95±9.38 | 91.90±6.75 | 83.81±11.71 | 65.71±12.46 | 75.24±17.51 | 83.33±6.04 | 93.81 ±6.37 |
| D10 | 32.51±5.45 | 93.46±1.90 | 14.29±2.62 | 93.59±1.32 | 67.88±5.03 | 64.11±4.15 | 93.51±2.17 | 80.56±4.58 | 88.83±2.25 | 53.42±9.31 | 92.60±2.50 | 93.98 ±2.84 |
| D11 | 51.53±2.47 | 48.61±1.87 | 0.00±0.00 | 57.43±2.38 | 41.10±2.49 | 36.65±3.28 | 60.76±2.52 | 60.76±1.01 | 50.41±3.30 | 34.16±4.59 | 53.16±2.75 | 61.74 ±1.84 |
| D12 | 67.38±11.61 | 67.14±22.64 | 35.71±12.94 | 69.05±14.89 | 41.67±12.26 | 67.86±21.77 | 64.29±10.29 | 57.86±11.93 | 91.61±7.36 | 69.44±7.86 | 64.52±24.37 | 66.19±25.09 |
| D13 | 79.46±22.51 | 84.46±10.58 | 34.64±18.75 | 81.79±13.30 | 50.36±22.87 | 71.25±18.22 | 85.89±15.07 | 75.00±17.58 | 83.81±11.16 | 59.68±6.84 | 91.61±9.89 | 91.61 ±9.89 |
| D14 | 18.24±11.23 | 93.62±6.48 | 68.43±13.64 | 81.79±12.27 | 69.31±18.04 | 68.07±11.06 | 88.50±9.73 | 79.48±12.47 | — | 87.83±4.78 | 93.64±6.98 | 93.64 ±6.98 |
| D15 | 30.88±10.02 | 49.34±13.62 | 6.76±6.89 | 9.85±7.92 | 40.51±7.62 | 49.01±9.88 | 29.56±15.31 | 32.02±9.95 | 59.34±13.57 | 44.22±5.62 | 56.76±11.23 | 56.76 ±11.23 |
| Ave | 44.41±8.37 | 78.47±5.88 | 29.8±6.28 | 65.98±5.54 | 53.07±8.23 | 58.64±7.78 | 74.82±6.48 | 70.30±7.71 | 75.46±5.90 | 51.04±5.11 | 81.89±5.72 | 83.93 ±5.71 |
| Rank | 8.40 | 4.73 | 11.13 | 5.67 | 8.47 | 7.87 | 4.93 | 6.53 | 5.60 | 9.07 | 3.67 | 1.47 |

TABLE III
CLASSIFICATION PRECISION(%) COMPARISON WITH OTHER 11 POPULAR METHODS ON 15 DATASETS

| No.s | MEEFI | FENN | ssODM | RF | WFCCL | M-FCCL | R-FCCL | GB-KNN | CM-CCL | GBCT | GB-CCLF | GB-CCL |
|------|--------------------|-------------|------------|--------------------|-------------|-------------|--------------------|--------------------|-------------|-------------|-------------|---------------------|
| D1 | 84.08 ±5.55 | 77.69±5.70 | 29.5±3.05 | 72.86±5.02 | 52.58±4.54 | 46.6±16.80 | 80.14±5.15 | 66.33±5.31 | 80.06±12.64 | 50.00±70.71 | 75.42±3.17 | 80.68±2.58 |
| D2 | 93.74±4.00 | 81.26±5.85 | 5.33±1.08 | 88.82±6.07 | 91.44±5.09 | 92.26±3.37 | 93.63±4.77 | 64.71±8.36 | 88.50±9.30 | 50.00±70.71 | 94.00±5.44 | 94.63 ±5.07 |
| D3 | 2.55±0.82 | 70.89±0.90 | 2.48±0.10 | 88.61±0.83 | 70.79±6.03 | 54.58±2.99 | 92.32 ±0.68 | 87.26±1.45 | 77.51±0.81 | 49.29±6.79 | 85.98±0.98 | 87.32±0.96 |
| D4 | 10.09±3.57 | 81.98±4.49 | 0.01±0.01 | 1.62±0.72 | 52.37±4.03 | 46.35±4.37 | 67.51±4.03 | 61.89±2.39 | 58.16±2.74 | 85.00±0.00 | 84.92±2.03 | 85.04 ±2.00 |
| D5 | 17.58±0.41 | 81.75±0.82 | 32.42±0.41 | 83.97 ±0.95 | 0.08±0.00 | 82.78±1.62 | 83.88±1.13 | 72.17±1.31 | 75.69±0.49 | 70.62±35.41 | 77.99±1.03 | 73.50±1.00 |
| D6 | 23.42±9.27 | 93.92±2.20 | 1.74±0.30 | 79.18±3.64 | 42.01±8.38 | 45.14±6.03 | 97.58±1.20 | 70.28±7.97 | 91.73±1.54 | 85.71±20.20 | 97.65±1.94 | 97.97 ±1.34 |
| D7 | 2.94±1.71 | 70.41±10.02 | 0.48±0.23 | 4.83±3.31 | 32.89±6.63 | 49.59±9.46 | 45.33±13.75 | 71.43±6.03 | 58.66±8.02 | 0.00±0.00 | 82.03±3.81 | 83.06 ±4.03 |
| D8 | 68.49±7.81 | 82.74±8.35 | 37.66±5.09 | 76.36±9.15 | 56.58±17.86 | 45.47±11.06 | 88.22±10.49 | 83.06±14.33 | 90.64±5.01 | 83.43±0.67 | 87.39±12.35 | 90.84 ±8.02 |
| D9 | 32.83±16.91 | 87.85±4.83 | 11.11±4.67 | 91.64±5.81 | 87.62±7.68 | 90.36±10.85 | 92.13±6.17 | 85.28±10.37 | 53.68±19.23 | 77.94±31.20 | 85.25±4.51 | 95.19 ±4.96 |
| D10 | 21.16±8.70 | 93.47±1.92 | 2.04±0.37 | 94.24±1.33 | 70.63±4.37 | 66.44±5.46 | 93.81±2.10 | 83.70±3.20 | 89.07±1.63 | 42.81±24.71 | 92.90±2.33 | 94.97 ±2.61 |
| D11 | 24.00±8.78 | 25.23±2.40 | 0.00±0.00 | 44.58±5.32 | 25.83±11.48 | 24.62±2.24 | 32.98±9.17 | 47.24 ±4.45 | 21.15±9.40 | 0.00±0.00 | 33.48±3.07 | 39.19±3.82 |
| D12 | 51.83±26.64 | 63.37±26.32 | 17.86±6.47 | 59.20±21.35 | 37.17±4.9 | 59.83±26.24 | 37.48±16 | 46.08±18.09 | 91.20±15.29 | 75.00±35.36 | 61.67±29.41 | 62.5±29.94 |
| D13 | 83.58±17.98 | 86.54±11.01 | 17.32±9.38 | 80.32±21.31 | 53.33±20.30 | 64.42±18.95 | 89.58±11.66 | 73.67±23.09 | 82.00±17.91 | 40.86±15.36 | 92.33±10.01 | 92.33 ±10.01 |
| D14 | 11.10±9.82 | 92.10±9.90 | 19.10±9.67 | 61.32±22.47 | 66.49±19.01 | 50.62±16.58 | 82.32±14.80 | 64.48±19.87 | — | 88.18±2.57 | 83.32±14.80 | 83.32 ±14.80 |
| D15 | 20.60±11.38 | 43.24±12.56 | 0.66±0.69 | 7.00±4.80 | 31.44±8.16 | 39.26±11.12 | 23.47±9.40 | 24.12±9.67 | 50.20±14.54 | 29.17±5.89 | 47.82±14.56 | 47.82 ±14.56 |
| Ave | 36.53±8.89 | 75.50±7.15 | 11.85±2.77 | 62.3±7.47 | 51.41±9.41 | 57.22±9.81 | 73.36±7.37 | 66.78±9.06 | 72.02±8.47 | 55.20±21.31 | 78.81±7.30 | 80.56 ±7.05 |
| Rank | 8.73 | 4.93 | 11.60 | 6.07 | 8.33 | 7.53 | 4.40 | 6.67 | 5.80 | 8.13 | 3.87 | 1.80 |

where $f(x, a)$ denotes the value of object x under feature a , $\max(f(a))$ and $\min(f(a))$ are the maximum and minimum values of all objects on feature a , respectively.

2) *Baselines*: To enhance the convincing experimental comparison, we compare GB-CCL with 11 representative state-of-the-art peers. These baselines include the multilayer ensemble evolving fuzzy inference method (MEEFI) [34], fuzzy edited nearest neighbor classifier (FENN) [35], semisupervised optimal margin distribution machine(ssODM) [36], random forest model (RF) [37], weighted fuzzy concept-cognitive learning (WFCCL) [38], memory-based CCL for fuzzy context (M-FCCL) [28], robust fuzzy-based concept-cognitive learning (R-FCCL) [39], GB-KNN classifier [8], concept-cognitive learning with collaborative multiscale (CM-CCL) [40], and clustering method with granular-ball (GBCT) [41]. In addition, five ablation experiments are carried out to illustrate the effectiveness of the similarity measures, GB generation, concept learning, and prediction mechanisms.

3) *Experimental Designs*: As introduced above, extensive experiments were conducted to evaluate the performance of the proposed GB-CCL model against competing methods. Specifically, to reduce the randomness of the evaluation results, tenfold cross-validation is employed for each dataset and the compared method for a fair comparison. All the parameter settings are consistent with their corresponding references. Meanwhile, five

evaluation measures, including Accuracy(“↑”), Precision(“↑”), Recall(“↑”), F1-score(“↑”), and Concept learning time(“↓”) are selected to make comparison from different perspectives, where “↑” and “↓” indicate “larger the better” and “smaller the better.” Moreover, to further illustrate the robustness of GB-CCL in a noise environment, each data is randomly added noise from 2% to 18% with a step of 2% for classification evaluation. All algorithms are implemented on a public computer with OS: Microsoft WIN10; Processor: Intel Core i7-6800K CPU 3.4 GHz× 12; Memory: 62.7 GB; Programming language: MATLAB2020a.

B. Performance Comparison (Rq.1)

To demonstrate the superiority of GB-CCL in the classification task, we compare it with 11 representative methods in this section. The detailed results are recorded in Tables II–V, where Ave denotes the average performance, and all the excellent results are bold. On D14, CM-CCL did not finish within the allowed runtime and is denoted as “—.” Meanwhile, for a clear comparison, average values and ranks of these compared methods are shown in Fig. 4. From these comparison results, we have three main observations as follows.

TABLE IV
CLASSIFICATION RECALL(%) COMPARISON WITH OTHER 11 POPULAR METHODS ON 15 DATASETS

| No.s | MEEFI | FENN | ssODM | RF | WFCCL | M-FCCL | R-FCCL | GB-KNN | CM-CCL | GBCT | GB-CCLF | GB-CCL |
|------|-------------|-------------|-------------|-------------------|-------------|-------------|-------------------|-------------|-------------|-------------|-------------|--------------------|
| D1 | 69.71±4.43 | 74.54±3.41 | 64.58±2.16 | 75.49±5.35 | 55.19±6.57 | 35.35±3.67 | 64.85±4.10 | 61.28±4.91 | 58.36±2.86 | 0.12±0.17 | 79.13±2.07 | 80.92±2.60 |
| D2 | 94.13±2.90 | 82.88±6.52 | 17.33±1.41 | 86.55±7.69 | 92.93±4.36 | 90.47±4.24 | 93.41±4.81 | 64.16±10.36 | 88.34±9.11 | 22.22±31.43 | 94.37±5.53 | 94.84±4.97 |
| D3 | 16.88±0.68 | 69.29±0.89 | 16.67±0.00 | 89.37±0.83 | 64.74±5.19 | 42.56±1.22 | 91.91±0.74 | 86.69±1.47 | 68.46±2.30 | 65.49±47.45 | 85.94±0.95 | 87.29±1.07 |
| D4 | 16.68±3.32 | 80.41±4.17 | 1.86±0.06 | 1.84±0.62 | 53.12±3.42 | 47.69±4.64 | 69.08±3.28 | 58.39±3.78 | 56.18±2.88 | 75.00±35.36 | 85.40±1.75 | 85.64±1.63 |
| D5 | 50.00±0.00 | 75.25±0.86 | 50.00±0.00 | 77.72±0.76 | 0.07±0.00 | 62.22±1.41 | 75.65±0.80 | 74.19±1.44 | 65.72±0.73 | 28.37±39.18 | 68.83±0.74 | 73.83±1.06 |
| D6 | 34.4±7.96 | 93.67±2.22 | 12.50±0.00 | 75.95±3.20 | 33.07±6.41 | 38.41±3.34 | 97.57±1.31 | 67.52±7.35 | 90.96±1.72 | 5.09±2.23 | 97.70±2.08 | 98.12±1.33 |
| D7 | 11.47±3.09 | 68.17±9.81 | 7.16±0.42 | 5.44±2.93 | 37.71±6.38 | 48.03±10.70 | 47.47±12.46 | 69.3±8.46 | 59.74±8.04 | 0.00±0.00 | 79.90±4.06 | 81.36±4.35 |
| D8 | 71.88±9.33 | 77.54±11.45 | 50.00±0.00 | 81.46±10.08 | 57.37±21.94 | 46.56±16.93 | 81.50±9.60 | 76.05±16.47 | 70.70±9.09 | 99.32±0.96 | 92.07±7.53 | 92.35±8.34 |
| D9 | 45.6±12.40 | 88.12±3.78 | 33.33±0.00 | 92.25±6.36 | 86.06±6.37 | 90.26±10.36 | 91.83±7.39 | 84.26±10.94 | 66.00±4.42 | 89.06±15.47 | 83.16±7.61 | 93.45±6.77 |
| D10 | 32.16±4.64 | 93.58±1.83 | 14.29±0.00 | 93.52±1.12 | 68.17±5.27 | 64.08±3.50 | 93.47±2.13 | 80.85±4.14 | 88.95±2.00 | 75.13±28.70 | 92.61±2.59 | 94.96±3.02 |
| D11 | 20.62±1.99 | 24.88±2.73 | 0.00±0.00 | 36.44±5.04 | 20.42±2.01 | 23.73±6.08 | 27.49±2.22 | 39.73±7.16 | 20.64±1.96 | 0.00±0.00 | 37.09±3.92 | 40.02±4.57 |
| D12 | 59.00±15.66 | 61.75±2.79 | 50.00±0.00 | 61.92±17.2 | 41.42±16.00 | 63.25±24.01 | 50.67±6.44 | 50.25±10.27 | 86.37±18.53 | 14.93±5.40 | 60.08±29.31 | 61.33±30.27 |
| D13 | 80.89±25.01 | 81.92±12.67 | 45.00±15.81 | 75.24±20.73 | 51.77±22.96 | 68.33±22.72 | 84.83±14.33 | 71.92±20.58 | 80.67±17.90 | 71.43±40.41 | 91.33±9.99 | 91.33±9.99 |
| D14 | 39.77±20.45 | 85.21±10.58 | 27.00±8.23 | 56.71±17.37 | 72.93±18.03 | 50.09±20.12 | 81.00±13.75 | 62.51±18.44 | - | 92.72±1.81 | 82.28±16.60 | 82.28±16.60 |
| D15 | 29.09±10.51 | 43.26±12.47 | 6.74±4.68 | 7.67±5.54 | 36.03±7.62 | 40.41±11.43 | 29.3±12.30 | 25.54±8.44 | 50.53±15.13 | 19.64±7.58 | 48.22±13.73 | 48.22±13.73 |
| Ave | 44.82±8.16 | 73.36±7.21 | 26.43±2.18 | 61.17±6.99 | 51.40±8.88 | 54.10±9.62 | 72.00±6.38 | 64.84±8.95 | 67.97±6.91 | 43.90±17.08 | 78.54±7.23 | 80.40±7.35 |
| Rank | 8.47 | 4.73 | 10.87 | 5.87 | 8.80 | 7.87 | 4.60 | 6.67 | 6.53 | 8.00 | 3.73 | 1.67 |

TABLE V
CLASSIFICATION F1-SCORE(%) COMPARISON WITH OTHER 11 POPULAR METHODS ON 15 DATASETS

| No.s | MEEFI | FENN | ssODM | RF | WFCCL | M-FCCL | R-FCCL | GB-KNN | CM-CCL | GBCT | GB-CCLF | GB-CCL |
|------|-------------|-------------|-------------|-------------------|-------------|-------------|-------------------|-------------------|-------------|-------------|-------------|--------------------|
| D1 | 76.17±4.45 | 76.01±3.83 | 40.44±3.27 | 74.12±4.97 | 53.79±5.20 | 39.41±8.15 | 71.60±3.72 | 63.55±3.80 | 67.25±6.59 | 22.98±0.2 | 77.21±2.45 | 80.78±2.40 |
| D2 | 93.91±3.20 | 81.99±5.67 | 8.12±1.38 | 87.62±6.63 | 92.16±4.56 | 91.32±3.32 | 93.51±4.73 | 64.11±8.20 | 88.39±9.06 | 42.67±5.49 | 94.18±5.47 | 94.73±5.00 |
| D3 | 4.39±1.18 | 70.08±0.88 | 4.32±0.15 | 88.99±0.83 | 67.55±5.07 | 47.82±1.82 | 92.12±0.70 | 86.98±1.44 | 72.69±0.94 | 23.57±6.84 | 85.96±0.94 | 87.31±1.00 |
| D4 | 12.38±3.72 | 81.18±4.26 | 0.03±0.01 | 1.70±0.67 | 52.72±3.62 | 46.94±4.08 | 68.27±3.55 | 60.06±2.93 | 57.13±2.53 | 19.73±0.39 | 85.16±1.84 | 85.34±1.77 |
| D5 | 26.01±0.45 | 78.36±0.82 | 39.33±0.30 | 80.72±0.78 | 0.07±0.00 | 71.04±1.51 | 79.55±0.90 | 73.17±1.36 | 70.36±0.63 | 49.53±13.31 | 73.13±0.80 | 73.66±1.02 |
| D6 | 27.56±9.16 | 93.80±2.16 | 3.05±0.46 | 77.53±3.36 | 36.75±6.70 | 41.30±3.47 | 97.58±1.21 | 68.81±7.40 | 91.34±1.5 | 11.71±7.49 | 97.67±1.99 | 98.05±1.33 |
| D7 | 4.56±2.36 | 69.20±9.68 | 0.90±0.40 | 4.95±2.94 | 34.92±6.07 | 48.68±9.90 | 46.28±13.12 | 70.27±6.96 | 59.09±7.68 | 7.38±5.23 | 80.92±3.63 | 82.16±4.02 |
| D8 | 70.06±8.19 | 79.3±6.05 | 42.78±3.37 | 78.71±9.00 | 56.59±19.64 | 45.83±13.65 | 84.50±9.00 | 79.03±14.35 | 79.62±6.96 | 73.26±0.92 | 89.41±9.51 | 91.54±7.84 |
| D9 | 37.12±16.4 | 87.96±4.04 | 16.16±5.42 | 91.92±5.88 | 86.81±6.92 | 90.31±10.60 | 91.94±6.60 | 84.76±10.63 | 57.97±11.9 | 71.43±23.19 | 84.11±5.83 | 94.29±5.79 |
| D10 | 24.83±7.65 | 93.52±1.86 | 3.56±0.58 | 93.88±1.16 | 69.37±4.79 | 65.19±4.09 | 93.64±2.10 | 82.24±3.63 | 89.01±1.78 | 51.04±9.95 | 92.75±2.45 | 95.02±2.80 |
| D11 | 21.69±4.65 | 25.05±2.54 | 0.00±0.00 | 40.05±4.99 | 22.26±5.67 | 23.79±2.90 | 29.52±4.47 | 42.86±4.62 | 20.33±5.1 | 17.43±1.08 | 35.18±3.38 | 39.59±4.16 |
| D12 | 53.78±21.02 | 62.32±25.31 | 25.68±7.45 | 60.18±18.92 | 37.28±16.15 | 61.17±25.04 | 42.19±11.37 | 47.36±14.32 | 88.57±16.75 | 52.62±1.28 | 60.75±29.38 | 61.78±30.11 |
| D13 | 81.52±22.51 | 83.85±10.74 | 24.30±11.52 | 77.35±20.97 | 52.25±21.56 | 66.19±20.46 | 87.36±12.40 | 72.65±21.62 | 81.10±17.3 | 50.31±7.23 | 91.63±9.21 | 91.63±9.21 |
| D14 | 16.81±13.02 | 90.50±9.71 | 22.16±9.28 | 58.67±19.29 | 69.44±18.42 | 49.37±16.42 | 82.93±13.92 | 63.29±18.73 | - | 64.91±27.05 | 82.74±15.65 | 82.74±15.65 |
| D15 | 23.45±10.32 | 42.88±12.09 | 1.17±1.16 | 7.14±4.99 | 33.47±7.76 | 39.72±11.07 | 25.87±10.41 | 24.24±8.16 | 50.26±14.77 | 39.71±6.40 | 47.92±13.89 | 47.92±13.89 |
| Ave | 38.28±8.55 | 74.40±6.64 | 15.47±2.98 | 61.57±7.03 | 51.03±8.81 | 55.21±9.10 | 72.46±6.55 | 65.56±8.54 | 69.51±7.39 | 39.89±7.74 | 78.58±7.09 | 80.44±7.07 |
| Rank | 8.87 | 4.53 | 11.60 | 5.93 | 8.27 | 7.53 | 4.33 | 6.40 | 6.13 | 9.27 | 3.53 | 1.53 |

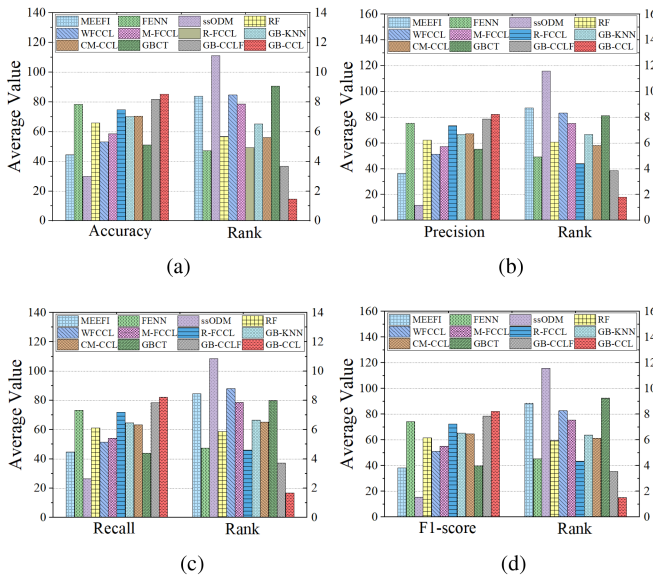


Fig. 4. Average results of GB-CCL and other 11 representative methods. (a) Accuracy. (b) Precision. (c) Recall. (d) F1-score.

1) The classification results of accuracy, precision, recall, and F1-score for the proposed GB-CCL method are obviously superior to other methods (i.e., MEEFI, FENN, ssODM, WFCCL,

CM-CCL, GBCT, and GB-CCLF) on the tested data. Concretely, the ranks of GB-CCL versus the selected comparison methods also show that our method has an obvious advantage in classification problems.

2) Note that the proposed GB-CCL method achieves the maximum average values and minimum rank of four experimental indexes on 15 datasets, illustrating its excellent performance in classification tasks.

3) Particularly, the ssODM is invalid for classification on the WiQW dataset and has poor performance in the Frogs dataset.

Moreover, to examine whether the classification performance differences among the compared methods are statistically significant, Friedman’s test [42] is employed at a significance level of $P = 0.05$. The null hypothesis states that all methods perform equally, which is rejected when the p -value is smaller than the predefined threshold. Specifically, the p -values for accuracy, precision, recall, and F1-score are 4.61×10^{-14} , 9.31×10^{-14} , 6.64×10^{-13} , and 3.40×10^{-16} , which is far lower than 0.05, indicating there has a significant difference among compared methods.

Based on the above results, Nemenyi’s post hoc test [43] is further conducted to assess the pairwise differences between GB-CCL and each baseline. The results are illustrated in Fig. 5. At the significance level of $P = 0.05$, the critical difference value is 4.3025 with $N_d = 15$ datasets and $N_m = 12$ methods.

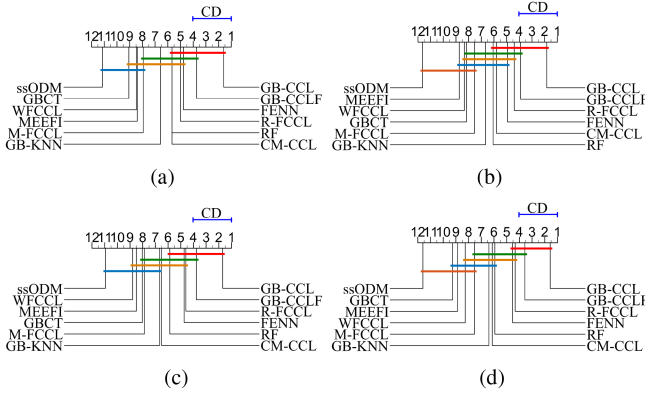


Fig. 5. Nemenyi test on GB-CCL and 11 representative methods. (a) Accuracy. (b) Precision. (c) Recall. (d) F1-score.

TABLE VI
CLASSIFICATION ACCURACY OF ABLATION EXPERIMENTS

| No.s | Ours _{cos} | Ours _{RBF} | Ours _{GB} | Ours _{Con} | Ours _{Pre} | Ours |
|------|---------------------|---------------------|--------------------|---------------------|---------------------|---------------------|
| D1 | 79.77±2.60 | 80.06±1.84 | 51.03±3.77 | 77.66±2.91 | 78.88±1.79 | 81.52 ±2.34 |
| D2 | 93.72±2.60 | 94.26±3.74 | 95.52±2.38 | 83.33±6.09 | 90.46±5.59 | 95.65 ±3.18 |
| D3 | 86.58±1.02 | 86.72±1.01 | 62.30±2.23 | 81.90±1.61 | 86.44±1.07 | 86.72 ±1.01 |
| D4 | 86.67 ±1.08 | 86.39±0.96 | 13.68±2.28 | 79.28±2.08 | 84.52±0.78 | 86.39±0.96 |
| D5 | 75.59±0.70 | 75.80±0.96 | 54.1±4.06 | 71.24±1.04 | 75.23±0.71 | 75.8 ±0.96 |
| D6 | 97.96±1.23 | 96.84±1.46 | 97.31±2.46 | 94.52±2.49 | 97.68±1.86 | 97.96 ±1.56 |
| D7 | 81.11±6.25 | 80.56±5.71 | 83.61±4.62 | 79.72±6.68 | 83.33±4.90 | 83.89 ±4.86 |
| D8 | 90.87±9.34 | 91.37±7.06 | 77.03±11.50 | 85.68±8.41 | 91.37±4.15 | 93.26 ±6.57 |
| D9 | 90.00±4.74 | 90.00±7.26 | 88.10±6.04 | 86.19±10.15 | 90.95±7.60 | 93.81 ±6.37 |
| D10 | 94.55 ±2.09 | 94.55 ±2.09 | 66.97±2.9 | 86.67±3.37 | 92.73±3.03 | 93.98±2.84 |
| D11 | 61.25±2.35 | 61.49±2.50 | 33.18±3.77 | 61.23±2.13 | 60.55±2.00 | 61.74 ±1.84 |
| D12 | 60.71±27.84 | 71.90±21.20 | 70.95±13.03 | 70.00±20.76 | 68.33±21.71 | 76.43 ±15.41 |
| D13 | 86.25±11.23 | 67.86±20.48 | 91.43±12.05 | 81.79±11.93 | 90.54±13.88 | 91.61 ±9.89 |
| D14 | 96.73±3.76 | 96.20±5.04 | 93.52±5.30 | 91.17±5.95 | 94.44±5.86 | 97.78 ±3.88 |
| D15 | 60.70±10.32 | 62.50±8.55 | 61.84±9.78 | 62.50±8.55 | 61.25±8.44 | 62.87 ±9.74 |
| Ave | 82.83±5.81 | 82.43±5.99 | 69.37±5.74 | 79.53±6.28 | 83.11±5.56 | 85.29 ±4.76 |
| Rank | 3.20 | 2.67 | 4.60 | 5.00 | 3.80 | 1.20 |

Note: Ours_{cos}, Ours_{RBF}, Ours_{GB}, Ours_{Con}, and Ours_{Pre} denote the ablation experiments about the similarity cosine measure, RBF measure, granular ball generation, concept construction, and prediction.

As shown in the figure, GB-CCL achieves the highest overall rank and is significantly better than the other 7, 6, 7, and 8 methods in terms of accuracy, precision, recall, and F1-score.

C. Differences Between the GB and Other CCL (Rq.2)

In this section, we provide a further analysis by comparing our method with GBC models and other CCL approaches in classification performance and concept learning time.

1) *Comparison of Classification Performance*: The classification results of comparing seven CCL and GBC methods are summarized in Tables II–V. Meanwhile, the classification accuracy of ablation experiments is also shown in Table VI. From these results, we have three main observations as follows.

a) Our proposed GB-CCL method achieves higher accuracy/precision/recall/F1-score than two GBC methods (GB-KNN and GBCT) and five CCL models (WFCCL, M-FCCL, R-FCCL, CM-CCL, and GB-CCLF) in most cases. Concretely, the ranks of GB-CCL versus the selected comparison methods also show that our method is obviously superior to others.

b) Compared with the GB-CCL without fuzzy concept, i.e., GB-CCLF, our method achieves better classification performance in most cases. These results also show the validity and

TABLE VII
CONCEPT LEARNING TIME(S) COMPARISON OF FIVE CCL METHODS

| Dataset | WFCCL | M-FCCL | R-FCCL | CM-CCL | GB-CCL |
|---------|-----------|----------|-------------|----------|----------------|
| D1 | 452.56 | 1066.80 | 282.31 | 552.36 | 28.26 |
| D2 | 16.91 | 16.70 | 6.06 | 102.78 | 0.78 |
| D3 | 18262.23 | 15577.00 | 3967.60 | 210.36 | 545.27 |
| D4 | 4494.16 | 1272.30 | 570.32 | 4540.00 | 233.00 |
| D5 | 123145.00 | 45901.85 | 23055 | 1088.60 | 2958.88 |
| D6 | 107.67 | 80.50 | 35.27 | 210.20 | 1.93 |
| D7 | 20.77 | 10.00 | 6.95 | 223.10 | 10.38 |
| D8 | 26.37 | 10.80 | 3.95 | 62.00 | 2.63 |
| D9 | 45.93 | 7.20 | 3.10 | 44.50 | 2.90 |
| D10 | 473.89 | 356.40 | 130.96 | 577.90 | 58.20 |
| D11 | 2611.55 | 3482.80 | 960.01 | 2368.40 | 204.25 |
| D12 | 0.83 | 0.83 | 5.82 | 45605.00 | 0.58 |
| D13 | 12.29 | 30.90 | 15.10 | 278.75 | 1.55 |
| D14 | 1.39 | 15.25 | 96.11 | – | 0.53 |
| D15 | 0.81 | 0.63 | 8.40 | 5917.30 | 0.56 |
| Ave | 9978.16 | 4522.00 | 435.14 | 4412.95 | 269.98 |

necessity of characterizing concepts from both positive and negative information perspectives in fuzzy decision formal contexts.

c) Compared with the other ablation experiments on similarity measure, GB generation, concept construction, and prediction, the proposed GB-CCL achieves the highest accuracy and lowest ranks, indicating the reasonability of the designed framework.

2) *Comparison of Concept Learning Time*: The results of comparing the concept learning time of GB-CCL with all the involved CCL methods are recorded in Table VII. From the comparison results, we have three observations as follows.

a) The proposed GB-CCL consumes the least concept learning time among all CCL methods except on the Move dataset of R-FCCL. The main reason is that our concept learning is based on justifiable GB generation space, and in the whole process, there is no parameter optimization involved.

b) WFCCL shows the worst performance, and CM-CCL and M-FCCL also show poor performance in most cases, mainly because they rely on neighborhood-based information granules and parameter optimization. Moreover, the introduction of attribute weights computation in W-FCCL increases computational costs.

c) GB-CCL achieves the lowest concept learning time and best classification performance among all the CCL methods, which indicates that GB-CCL has a better concept learning performance than the current state-of-the-art CCL methods.

D. Influence of the Proposed Viewpoint (Rq.3)

According to the above analysis, we know that GB-CCL is without any parameters, and the process of GB generation is an adaptive learning based on the fuzzy decision formal context. Hence, in order to verify the robustness of the proposed ideas, we add noise to the raw conditional attributes to show the changes in the classification performance of GB-CCL under different noise situations. Specifically, random noise is introduced from 2% to 18% with an increment of 2%. For each noise level, v random numbers (g_1, g_2, \dots, g_v) are generated within the range of $[-g, g]$, where g denotes the random error threshold. The method of adding noise is as follows:

$$\tilde{I}_i(x, a) = \begin{cases} \tilde{I}(x, a) + g_i, & \text{if } (0 \leq \tilde{I}(x, a) + g_i \leq 1) \\ \tilde{I}(x, a), & \text{else} \end{cases}$$

TABLE VIII
CLASSIFICATION ACCURACY(%) COMPARISON OF GB-CCL ON DIFFERENT NOISE LEVELS

| Dataset | RAW | Noise Level | | | | | | | | |
|---------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| | | 2% | 4% | 6% | 8% | 10% | 12% | 14% | 16% | 18% |
| D1 | 81.52±2.34 | 82.31±1.47 | 80.66±3.99 | 80.38±3.66 | 80.62±2.99 | 79.91±2.76 | 79.44±3.03 | 79.58±2.42 | 78.93±2.18 | 78.79±1.43 |
| D2 | 95.65±3.18 | 95.56±2.99 | 95.55±3.26 | 96.64±2.57 | 94.13±4.07 | 94.14±2.75 | 93.32±4.37 | 93.60±2.92 | 93.87±3.40 | 95.27±3.94 |
| D3 | 86.72±1.01 | 85.69±0.99 | 85.36±1.02 | 85.63±0.66 | 85.43±1.13 | 85.63±0.81 | 85.47±1.17 | 85.14±0.86 | 85.06±0.77 | 84.88±1.00 |
| D4 | 86.39±0.96 | 85.55±1.51 | 85.06±0.98 | 85.31±1.24 | 84.92±1.08 | 84.77±0.84 | 84.60±1.54 | 84.06±1.04 | 84.73±0.74 | 83.70±1.21 |
| D5 | 75.80±0.96 | 75.75±1.14 | 75.80±0.73 | 76.23±0.81 | 76.58±0.65 | 76.05±1.12 | 76.27±0.92 | 76.19±0.90 | 76.15±0.93 | 76.06±0.74 |
| D6 | 97.96±1.56 | 80.56±7.17 | 78.89±8.30 | 82.22±10.49 | 77.22±9.60 | 78.61±7.53 | 80.28±7.46 | 79.44±9.18 | 79.72±7.41 | 79.72±10.15 |
| D7 | 83.89±4.86 | 97.03±1.68 | 94.80±2.02 | 94.43±1.90 | 93.50±1.58 | 92.01±1.76 | 88.22±3.00 | 86.36±3.55 | 85.61±2.39 | 84.40±3.10 |
| D8 | 93.26±6.57 | 89.79±6.34 | 89.71±5.91 | 92.79±7.42 | 92.84±5.48 | 91.34±5.30 | 90.26±7.11 | 90.26±6.98 | 90.71±9.18 | 90.79±7.54 |
| D9 | 93.81±6.37 | 89.52±5.85 | 90.95±6.53 | 89.05±7.79 | 88.57±9.58 | 91.43±7.03 | 89.05±7.79 | 92.38±6.81 | 90.95±6.90 | 91.43±5.85 |
| D10 | 93.98±2.84 | 94.20±1.83 | 93.72±1.91 | 93.98±1.46 | 92.55±1.06 | 93.25±1.02 | 93.77±1.84 | 93.94±2.24 | 94.68±1.73 | 93.77±1.46 |
| D11 | 61.74±1.84 | 61.41±1.44 | 59.94±2.06 | 60.45±2.11 | 59.37±2.54 | 59.02±2.38 | 59.39±1.50 | 59.41±1.41 | 58.96±2.34 | 58.47±1.84 |
| D12 | 66.19±25.09 | 66.19±25.09 | 72.62±7.63 | 74.52±16.84 | 72.62±17.47 | 69.29±12.32 | 69.05±16.84 | 75.71±16.14 | 67.62±15.84 | 75.71±14.10 |
| D13 | 91.61±9.89 | 80.89±12.66 | 82.14±15.82 | 77.86±15.13 | 79.29±16.24 | 84.82±13.92 | 80.71±16.43 | 85.89±15.07 | 74.82±11.43 | 84.82±13.92 |
| D14 | 93.64±6.98 | 92.12±6.69 | 93.07±4.86 | 91.10±8.12 | 93.52±6.69 | 91.07±7.79 | 93.05±9.47 | 91.19±6.85 | 91.10±6.98 | 91.10±6.98 |
| D15 | 56.76±11.23 | 60.55±13.98 | 60.55±12.18 | 58.2±13.13 | 57.65±10.55 | 58.75±15.18 | 61.84±11.86 | 61.14±12.18 | 58.75±12.67 | 61.18±10.92 |
| Ave | 83.93±5.71 | 82.47±6.06 | 82.59±5.15 | 82.59±6.22 | 81.92±6.05 | 82.01±5.50 | 81.65±6.29 | 82.29±5.90 | 80.78±5.66 | 82.01±5.61 |

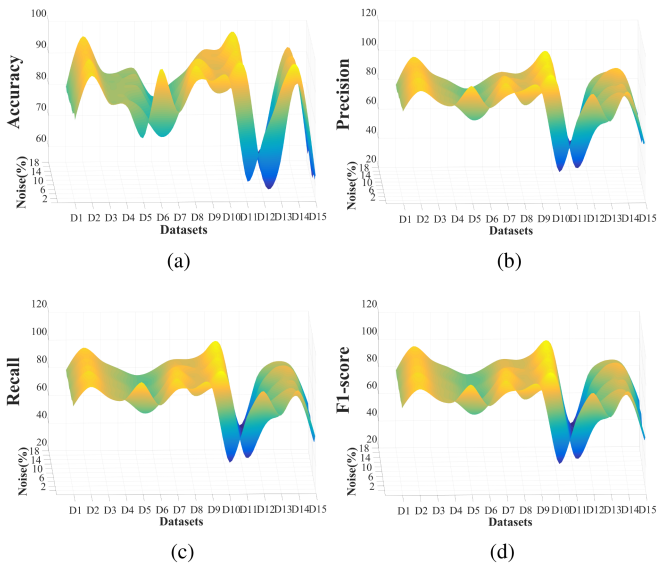


Fig. 6. Classification results comparison on different noise levels. (a) Accuracy. (b) Precision. (c) Recall. (d) F1-score.

where $\tilde{I}(x, a)$ represents the value of x under a in fuzzy decision formal context $FC = (G, M, \tilde{I}, D, J)$, $\tilde{I}_i(x, a)$ represents the value of x under a by add i th noise.

The classification accuracy of GB-CCL under different noise data levels is shown in Tables VIII. It can be seen from these tables and figures that the classification measures of GB-CCL fluctuate slightly in experimental datasets as the noise data increases. The more detailed comparison in accuracy, precision, recall, and F1-score is shown in Fig. 6. To further observe the variation induced by noise, the Box-plot is employed to show the distribution central location and a spread range of GB-CCL under different noise data levels, which is shown in Fig. 7. In these figures, the \square , $-$, \perp , and \top represent the mean value, median line, first quartile, and third quartile, respectively. The difference between \perp and \top is the quartile, which is also called the 1.5 interquartile range (1.5 IQR). The data outside the IQR are called an outlier.

All these measures could be used to analyze the differences among the compared methods. The compared methods are considered the same when the adopted measures differ little.

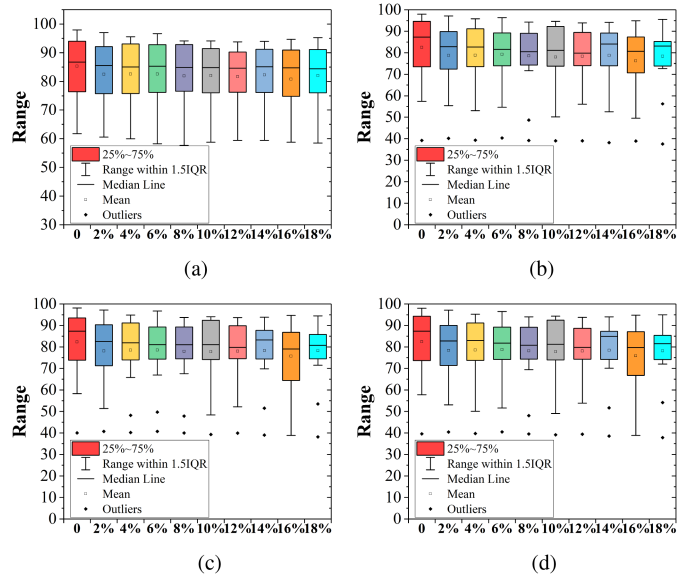


Fig. 7. Box diagram of GB-CCL on different noise data levels. (a) Accuracy. (b) Precision. (c) Recall. (d) F1-score.

From Fig. 7, we could obtain the following point: 1) at different noise levels, the median line and mean values of the GB-CCL method are almost on the same level, indicating that the central location consistency of GB-CCL with noise data increases and 2) except for the WiQW dataset, other results are all in the 1.5 IQR, and the 1.5 IQR is roughly the same in most situations. This robustness can be attributed to two key components of the GB-CCL framework. First, the adaptive granulation generation mechanism improves local noise tolerance by controlling purity, thereby preventing the influence of noisy samples on the local data structure. Moreover, the fuzzy three-way concept learning method further restricts the scope of concept learning by integrating positive and negative context information. They all collaborate to ensure GB-CCL's stable and robust learning performance in noisy environments.

VI. CONCLUSION

Two influential studies in GrC and artificial intelligence, namely, CCL and GBC, can achieve the same results through

different emphases. The proposed GB-CCL is a fruitful marriage of GBC and CCL to study uncertain information and knowledge processing embedded in complex data. It can achieve effective, robust, and adaptive concept learning for knowledge discovery. In this article, the theoretical framework of GB-CCL has been fully developed, and the corresponding experiments have been thoroughly demonstrated.

Despite the proposed GB-CCL method showing good performance on knowledge discovery tasks, we acknowledge a range of limitations to our method thus far. For example, we have not yet tested a wider range of complex problem application scenarios, including cognitive and learning multimodal scenarios with various data types. The granular representation in GB-CCL provides a flexible mechanism for constructing concept intentions at different granularities, which offers a natural foundation for extending the framework to multimodal concept learning. In addition, another interesting issue for future work is to study CCL in an open environment, namely, environments that change over time. Owing to its dynamic concept learning mechanism and adaptive granular construction, GB-CCL is inherently suitable for incrementally updating concepts and accommodating drifting patterns. Finally, future work will involve exploring concept-cognitive mechanisms, optimizing cognitive systems with various fuzzy evaluation measures, and systematically modeling cognitive processes, particularly in large-scale, dynamic data environments.

ACKNOWLEDGMENT

The authors would like to thank the Associate Editor and the reviewers for their insightful comments and suggestions.

REFERENCES

- [1] B. M. Lake, R. Salakhutdinov, and J. B. Tenenbaum, "Human-level concept learning through probabilistic program induction," *Science*, vol. 350, no. 6266, pp. 1332–1338, 2015.
- [2] K. Yuan, D. Miao, W. Ding, W. Pedrycz, and Y. Yao, "Robust semi-supervised feature selection with multi-granularity zentropy modeling," *IEEE Trans. Pattern Anal. Mach. Intell.*, early access, Dec. 24, 2025, doi: [10.1109/TPAMI.2025.3647921](https://doi.org/10.1109/TPAMI.2025.3647921).
- [3] X. Xin, H. Yu, Z. Xue, C. Wang, and T. Taleb, "A novel fuzzy concept-cognitive learning model with attribute fluctuation and concept clustering," *IEEE Trans. Fuzzy Syst.*, vol. 33, no. 10, pp. 3570–3581, Oct. 2025.
- [4] W. Li, B. Yang, W. Pedrycz, C. Zhang, and T. Zhan, "Adaptive hyper-box granulation with justifiable granularity for feature selection," *IEEE Trans. Knowl. Data Eng.*, vol. 37, no. 12, pp. 6847–6862, Dec. 2025.
- [5] K. Yuan, D. Miao, W. Pedrycz, W. Ding, and H. Zhang, "Ze-HFS: Zentropy-based uncertainty measure for heterogeneous feature selection and knowledge discovery," *IEEE Trans. Knowl. Data Eng.*, vol. 36, no. 11, pp. 7326–7339, Nov. 2024.
- [6] L. Chen, "Topological structure in visual perception," *Science*, vol. 218, no. 4573, pp. 699–700, 1982.
- [7] S. Xia et al., "Ball k -means: Fast adaptive clustering with no bounds," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 44, no. 1, pp. 87–99, Jan. 2022.
- [8] S. Xia et al., "Granular ball computing classifiers for efficient, scalable and robust learning," *Informat. Sci.*, vol. 483, pp. 136–152, 2019.
- [9] S. Xia et al., "GRRS: Accurate and efficient neighborhood rough set for feature selection," *IEEE Trans. Knowl. Data Eng.*, vol. 35, no. 9, pp. 9281–9294, Sep. 2023.
- [10] S. Xia, X. Dai, G. Wang, X. Gao, and E. Giem, "An efficient and adaptive granular-ball generation method in classification problem," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 35, no. 4, pp. 5319–5331, Apr. 2024.
- [11] X. Cao, X. Yang, S. Xia, G. Wang, and T. Li, "Open continual feature selection via granular-ball knowledge transfer," *IEEE Trans. Knowl. Data Eng.*, vol. 36, no. 12, pp. 8967–8980, Dec. 2024.
- [12] W. Li et al., "Granular-ball regeneration clustering with principle of justifiable granularity," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 36, no. 10, pp. 18173–18187, Oct. 2025.
- [13] D. Guo and W. Xu, "Fuzzy-based concept-cognitive learning: An investigation of novel approach to tumor diagnosis analysis," *Informat. Sci.*, vol. 639, 2023, Art. no. 118998.
- [14] Y. Ding and W. Xu, "Multi-granularity interval-intent fuzzy concept-cognitive learning: An attention-enhanced adaptive clustering framework," *Informat. Fusion*, vol. 127, 2026, Art. no. 103715.
- [15] D. Guo et al., "Concept-cognitive learning survey: Mining and fusing knowledge from data," *Informat. Fusion*, vol. 109, 2024, Art. no. 102426.
- [16] Y. Yao, "Interpreting concept learning in cognitive informatics and granular computing," *IEEE Trans. Syst., Man, Cybern., Part B-Cybern.*, vol. 39, no. 4, pp. 855–866, Aug. 2009.
- [17] W. Xu, D. Guo, J. Mi, Y. Qian, K. Zheng, and W. Ding, "Two-way concept-cognitive learning via concept movement viewpoint," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 34, no. 10, pp. 6798–6812, Oct. 2023.
- [18] J. Li et al., "Concept learning via granular computing: A cognitive viewpoint," *Informat. Sci.*, vol. 298, pp. 447–467, 2015.
- [19] Y. Shi, Y. Mi, J. Li, and W. Liu, "Concept-cognitive learning model for incremental concept learning," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 2, pp. 809–821, Feb. 2021.
- [20] J. Niu et al., "Parallel computing techniques for concept-cognitive learning based on granular computing," *Int. J. Mach. Learn. Cybern.*, vol. 9, no. 11, pp. 1785–1805, 2018.
- [21] Y. Shi et al., "Concurrent concept-cognitive learning model for classification," *Informat. Sci.*, vol. 496, pp. 65–81, 2019.
- [22] T. Zhang et al., "Incremental concept-cognitive learning based on attribute topology," *Int. J. Approx. Reasoning*, vol. 118, pp. 173–189, 2020.
- [23] X. Deng et al., "Uncertain multi-conceptual information acquisition and fusion for hierarchical classification," *Informat. Fusion*, vol. 125, 2025, Art. no. 103421.
- [24] Y. Lin et al., "Concept reduction via global relevance and redundancy viewpoints," *Informat. Sci.*, vol. 714, 2025, Art. no. 122205.
- [25] Y. Mi, Y. Shi, J. Li, W. Liu, and M. Yan, "Fuzzy-based concept learning method: Exploiting data with fuzzy conceptual clustering," *IEEE Trans. Cybern.*, vol. 52, no. 1, pp. 582–593, Jan. 2022.
- [26] Y. Mi, W. Liu, Y. Shi, and J. Li, "Semi-supervised concept learning by concept-cognitive learning and concept space," *IEEE Trans. Knowl. Data Eng.*, vol. 34, no. 5, pp. 2429–2442, May 2022.
- [27] W. Xu, D. Guo, Y. Qian, and W. Ding, "Two-way concept-cognitive learning method: A fuzzy-based progressive learning," *IEEE Trans. Fuzzy Syst.*, vol. 31, no. 6, pp. 1885–1899, Jun. 2023.
- [28] D. Guo et al., "M-FCCL: Memory-based concept-cognitive learning for dynamic fuzzy data classification and knowledge fusion," *Informat. Fusion*, vol. 100, 2023, Art. no. 101962.
- [29] D. Guo, W. Xu, Y. Qian, and W. Ding, "Fuzzy-granular concept-cognitive learning via three-way decision: Performance evaluation on dynamic knowledge discovery," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 3, pp. 1409–1423, Mar. 2024.
- [30] J. Wang, W. Xu, W. Ding, and Y. Qian, "Multiview fuzzy concept-cognitive learning with high-order information fusion of fuzzy attributes," *IEEE Trans. Fuzzy Syst.*, vol. 32, no. 12, pp. 6965–6978, Dec. 2024.
- [31] Z. Liu et al., "Multi-level information fusion for missing multi-label learning based on stochastic concept clustering," *Informat. Fusion*, vol. 115, 2025, Art. no. 102775.
- [32] K. Yuan et al., "An incremental learning mechanism for object classification based on progressive fuzzy three-way concept," *Informat. Sci.*, vol. 584, pp. 127–147, 2022.
- [33] W. Zhang, J. Ma, and S. Fan, "Variable threshold concept lattices," *Informat. Sci.*, vol. 177, no. 22, pp. 4883–4892, 2007.
- [34] X. Gu and Q. Shen, "Self-organizing divisive hierarchical Voronoi tessellation-based classifier," *Informat. Sci.*, vol. 603, pp. 106–129, 2022.
- [35] M. Yang and C. Cheng, "On the edited fuzzy K-nearest neighbor rule," *IEEE Trans. Syst., Man, Cybern.: Cybern.*, vol. 28, no. 3, pp. 461–466, Jun. 1998.
- [36] T. Zhang and Z. Zhou, "Optimal margin distribution machine," *IEEE Trans. Knowl. Data Eng.*, vol. 32, no. 6, pp. 1143–1156, Jun. 2020.
- [37] Z. Zhou, *Machine Learning*. Berlin, Germany: Springer, 2021.
- [38] C. Zhang et al., "Incremental concept-cognitive learning approach for concept classification oriented to weighted fuzzy concepts," *Knowl. Based Syst.*, vol. 260, 2023, Art. no. 110093.
- [39] D. Guo and W. Xu, "R-FCCL: An approach of fuzzy-based concept-cognitive learning with robustness for high-dimensional data," *J. Comput. Res. Develop.*, vol. 62, no. 2, pp. 383–396, 2025.

- [40] W. Xu, J. Wang, and Q. Zhang, "CM-CCL: Collaborative multi-scale concept-cognitive learning for knowledge discovery," *Pattern Recognit.*, vol. 171, 2026, Art. no. 112268.
- [41] S. Xia, B. Shi, Y. Wang, J. Xie, G. Wang, and X. Gao, "GBCT: Efficient and adaptive clustering via granular-ball computing for complex data," *IEEE Trans. Neural Netw. Learn. Syst.*, vol. 36, no. 7, pp. 12159–12172, Jul. 2025.
- [42] M. Friedman, "The use of ranks to avoid the assumption of normality implicit in the analysis of variance," *J. Amer. Stat. Assoc.*, vol. 32, pp. 675–701, 1937.
- [43] P. B. Nemenyi, "Distribution-free multiple comparisons," Ph.D. dissertation, Princeton Univ., Princeton, NJ, USA, 1963.



Doudou Guo received the M.Sc. degree in statistics and computer science from the School of Computer Science and Information Engineering, Harbin Normal University, Harbin, China, in 2021, and the Ph.D. degree in statistics and computer science from the College of Artificial Intelligence, Southwest University, Chongqing, China, in 2025.

He is currently an Associate Professor with the China University of Mining and Technology, Xuzhou, China. His current research interests include concept learning, granular computing, and fuzzy sets.



Weihua Xu received the M.Sc. degree in mathematics from the School of Mathematics and Information Sciences, Guangxi University, Nanning, China, in 2004, and the Ph.D. degree in mathematics from the School of Sciences, Xi'an Jiaotong University, Xi'an, China, in 2007.

He is currently a Professor with the College of Artificial Intelligence, Southwest University, Chongqing, China. He serves in the editorial boards of several international journals, and has authored or coauthored five monographs and more than 260 articles in international journals. His current research interests include granular computing, cognitive computing, and information fusion.



Shuyin Xia (Senior Member, IEEE) received the B.S. and M.S. degrees in computer science from the Chongqing University of Technology, Chongqing, China, in 2008 and 2012, respectively, and the Ph.D. degree from the College of Computer Science, Chongqing University, Chongqing, in 2015.

He is currently a Professor with the College of Computer Science and Technology, Chongqing University of Posts and Telecommunications, Chongqing, where he is also the Executive Deputy Director of the Big Data and Network Security Joint

Laboratory. He has authored or coauthored more than 30 articles in journals and conferences, including *IEEE TRANSACTIONS ON KNOWLEDGE AND DATA ENGINEERING*, *IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE*, *IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS*, *IEEE TRANSACTIONS ON FUZZY SYSTEMS*, and *Information Sciences*. His research outcomes, published as first or corresponding author in top-tier artificial intelligence journals and conferences, such as *IEEE TRANSACTIONS ON PATTERN ANALYSIS AND MACHINE INTELLIGENCE*, *IEEE TRANSACTIONS ON KNOWLEDGE AND DATA ENGINEERING*, *IEEE TRANSACTIONS ON IMAGE PROCESSING*, *ICML*, *AAAI*, *IJCAI*, and *ICDE*, have attracted follow-up studies by scholars from more than 100 world-renowned research institutions. His research interests include classifiers and granular computing.

Dr. Xia is the Vice Chairman of the Chongqing Association for Artificial Intelligence and has been recognized as a National-Level Young Talent, Chongqing Outstanding Young Scholar, and Chongqing Talent. He ranks among the top 2% of the world's top scientists. He was the recipient of the Second Prize of the National Teaching Achievement Award, First Prize of the Chongqing Natural Science Award, First Prize of the CCF Natural Science Award, and the Wu Wenjun Artificial Intelligence Science and Technology Progress Award, for his work.



Weiping Ding (Senior Member, IEEE) received the Ph.D. degree in computer science, Nanjing University of Aeronautics and Astronautics, Nanjing, China, in 2013.

From 2014 to 2015, he was a Postdoctoral Researcher with the Brain Research Center, National Chiao Tung University, Hsinchu, Taiwan. In 2016, he was a Visiting Scholar with the National University of Singapore, Singapore. From 2017 to 2018, he was a Visiting Professor with the University of Technology Sydney, Ultimo, NSW, Australia. He is currently a

Professor with Nantong University, Nantong, China. He has authored or coauthored more than 430 articles, including more than 220 IEEE Transactions papers. His 20 authored or coauthored papers have been selected as ESI Highly Cited Papers. His research interests include granular data mining and multimodal machine learning.

Dr. Ding is an Associate Editor/Area Editor/Editorial Board member of more than ten international prestigious journals, such as *IEEE TRANSACTIONS ON NEURAL NETWORKS AND LEARNING SYSTEMS*, *IEEE TRANSACTIONS ON FUZZY SYSTEMS*, *IEEE/CAA JOURNAL OF AUTOMATICA SINICA*, *IEEE TRANSACTIONS ON EMERGING TOPICS IN COMPUTATIONAL INTELLIGENCE*, *IEEE TRANSACTIONS ON INTELLIGENT TRANSPORTATION SYSTEMS*, *Information Fusion*, *Neurocomputing*, and *Applied Soft Computing*. He was the Leading Guest Editor of Special Issues in several prestigious journals, including *IEEE TRANSACTIONS ON EVOLUTIONARY COMPUTATION*, *IEEE TRANSACTIONS ON FUZZY SYSTEMS*, and *Information Fusion*.



Yuhua Qian (Member, IEEE) received the M.S. and Ph.D. degrees in computers with applications from Shanxi University, Taiyuan, China, in 2005 and 2011, respectively.

He is currently the Director with the Institute of Big Data Science and Industry, Shanxi University, Taiyuan, China, where he is also a Professor with the Key Laboratory of Computational Intelligence and Chinese Information Processing, Ministry of Education. He is best known for multigranulation rough sets in learning from categorical data and granular

computing. He has authored or coauthored more than 100 articles in his research field, which include machine learning, pattern recognition, granular computing, and artificial intelligence in international journals.



Kehua Yuan received the M.Sc. degree in computer science and technology from the College of Artificial Intelligence, Southwest University, Chongqing, China, in 2022. She is currently working toward the Ph.D. degree in computer science and technology with the School of Computer Science and Technology, Tongji University, Shanghai, China.

Her current research interests include granular computing and uncertainty analysis. She has authored over 10 articles on these topics in international journals.